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Informational Content of Equivalence Scales based on Minimum Needs Income

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ABSTRACT

Uniform equivalence scales are routinely used for welfare comparisons and require that utility function is IB/ESE (independent of base / equivalent scale exact). This condition itself requires restrictions on the level of measurability and interpersonal comparability of preferences across households, so called informational basis, in that welfare ordering must be Ordinal and Fully Comparable (OFC). We show that if one calculates equivalence scale at particular utility level, for example households living in poverty, the informational basis is much weaker and requires full comparability only at a single point. For this purpose we introduce the axiom of Ordinal Local Comparability (OLC) and show that equivalence scale based on Minimum Needs Income is consistent with that axiom. We argue that subjective equivalence scale using the intersection method offers practical application of equivalence scale consistent with OLC.

KEY WORDS: Informational Basis, Poverty, Equivalence Scales, Subjective Poverty Line, Local-comparability, Intersection Method

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1. Introduction

The proper derivation and use of equivalence scales is a key element in conducting an appropriate analysis of inequality, poverty, or welfare in a particular society. Well-defined equivalence scales can adjust for economies of scale within the family and thus allow for cross-household and cross-person comparisons (Coutler at al., 1992). Because equivalence scale involves adjustment of income so that households of different types can achieve the same level of utility, each approach to deriving equivalence scale implicitly assumes certain properties of the utility function and social welfare functional. Of particular importance are conditions required for effective comparisons between households which are summarized by the informational basis of a welfare ordering. The objective in this paper is to analyze the informational content underlying the equivalent scales calculated for a single group of households at a particular level of utility and characterized by the same minimum needs income.

For practical reasons the most common approach to using equivalence scales assumes applying the same required adjustment of income across all households with different utility levels (henceforth, uniform equivalence scales). Lewbel (1989) and Blackcorby and Donaldson (1991, 1993) independently showed that in order to use such uniform equivalence scale the household preferences must be consistent with IB/ESE (independent of base / equivalent scale exactness) condition. However, IB/ESE requires that the information structure supports inter-household comparisons of utility levels and thus welfare profiles must be Ordinal and Full Comparable (OFC). Such condition imposes restrictions on the preferences which may be difficult to satisfy or even verify.

Because OFC requires information about utility levels for members of the household, which is normally unobservable, Blackcorby and Donaldson (1993) proposes a practical solution where one can derive equivalence scale for a single reference group. It allows the consideration of only households at the same welfare level, without the need to know the value of utility at that level. Another benefit relates to aiding identification of the demand system (Blundell and Lewbel, 1991).

One example of equivalence scale derived for a single reference group is the subjective scale based on a Minimum Income Needs Question (MINQ), and the intersection method proposed by Goedhart et al. (1977). It explicitly focuses on households in poverty and does not impose any restrictions on the preferences. In fact, Hartog (1988) observed that in order to effectively use MINQ one only needs to assume that "people (…) associate a certain common, interpersonally comparable feeling of welfare with verbal description of the minimum level ('enough to get along')" (Hartog 1988, p. 255, citing Hagenaars 1986). Therefore, he concludes that "the cardinality of the welfare function is not required" and thus the demand functions for consumer goods are not confined to any particular shape (Hartog 1988, p. 252).

Even though the estimates of subjective equivalence scales based on MINQ and on the intersection method vary across studies, they consistently indicate that households require more income with larger size to reach the same welfare level. This validates the MINQ approach which is able to account for economies of scale within the household. With the use of large datasets, detailed demographic information, and careful econometric implementation, the intersection method can yield robust and economically relevant results. For example, Bishop *el al.* (2014) showed that a subjective equivalence scale can account for the differences among welfare states within Euro Zone countries, and that the marginal cost of the first child is high but then sharply declines.

We show that calculating an equivalence scale at a single utility level requires Ordinal Local Comparability (OLC) and puts minimal restrictions on household preferences. Further, we show that the subjective scale based on MINQ only requires OLC and can be regarded as theoretically sound and robust to most possible utility functions, and yet empirically viable and identifiable.

The paper is organized as follows. Section 2 introduces axiom of *ordinality and local comparability* (OLC), Section 3 presents assumptions underlying MINQ-based subjective equivalence scales derived by using the intersection method and shows that they only require OLC, and Section 4 concludes.

2. Ordinal and Local Comparable Informational basis

The most common approach to equivalence scales is what we call uniform equivalence scales, where equivalence scales in are assumed to be constant across households for all levels of utility across households. It means that some level of intra- and inter-households comparability has to be assumed. This takes us back to Sen's *informational basis* of household profiles (Sen, 1974), which is defined by a set of restrictions on the properties of the household utility profiles and in the ability to make meaningful intra- and inter-households comparability.

Typically there are two criteria defining *information basis*. The first one distinguishes *ordinal* versus *cardinal* preferences. It relates to the ability to make meaningful intra-households comparisons of utility. The second criterion specifies *non-comparability* vs. *full-comparability* of the household utility profiles and relates to the ability of making meaningful inter-households comparisons of utility.[3]

[3] By intra-household comparisons we mean comparing the utility level for the same household at different levels of income and prices.

According to the first criterion, *ordinal informational basis* provides settings where pairwise comparisons of utility over different states are not affected by *monotone* transformation of the individual utility function. *Cardinal informational basis* typically restricts the invariance to *affine* transformations.

Under the second criterion, pairwise comparisons of utility across households are affected by different monotone transformations of the household utility functions. If these transformations are allowed, we are then in the Arrowian *non-comparability informational basis*, where inter-households comparison are not allowed. Only if the same monotone transformation is required for all households, we are in the *full-comparability informational basis*.[4]

Because these restrictions inform the social planner about available intra-household or inter-household utility comparisons, modern positive theory formalizes the restrictions imposed on the social welfare functionals, and eventually on the profiles of household's utility or on the equivalence scales, by using *invariance axioms*. These conditions guarantee that social preference orderings are insensitive to transforms of the utility profiles (D'Aspremont Claude and Louis Gevers, 2002).

Under one extreme case, the Arrowian *ordinal non-comparability* (ONC), Arrow (1963), no equivalence scales can be deduced. In fact, for uniform equivalence scales to be meaningful a much stronger information basis is required. It has been described by Blackorby and Donaldson (1993) as "*ordinal full-comparability plus*", and by Lewbel (1989) as "*ordinal level comparability.*" Both share the same characteristics in that comparability for all households must be allowed, and thus we call this case *ordinal full-comparability* (OFC).

[4] Note that because any monotone transformation is permitted, we are still in an *ordinal* and *full-comparable* setting (D'Aspremont and Gevers, 1977, 2002)

In this section we investigate if there is a weaker ordinal informational basis consistent with more general non-uniform equivalence scales. Therefore, we introduce new axiom of *ordinality and local-comparability* (OLC), which is an intermediate case between the axioms of ONC and OFC. The only relevant information is contained in utility comparison for households who have the same level of well-being. We present an example of households having income equal to poverty line.

2.1. Notation

Assume a population of N households, characterized by two attributes: the total actual income of the household, $y \epsilon \mathbb{R}_{+}$, and the level of needs captured by a set of welfare-relevant "non-income" characteristics, α (i.e., household size and composition). Suppose we partition the population into n disjoint groups, according to their level of needs and that the needs can be ranked, $\alpha = 1,2, \dots, n$ $(1 \leq n \leq N)$. Thus, if $\alpha > \bar{\alpha}$ then the level of needs in the group α is greater than that in group $\bar{\alpha}$. The set of all possible characteristics is denoted by Γ with cardinality n. The only difference within any subgroup is actual income, y.

Further, let us assume that differences in needs are incorporated in the indirect utility functions $\mathrm{V}: \mathbb{R}_{++}^{m+1} \cup \Gamma \rightarrow \mathbb{R}$, whose typical image $V(p, y, \alpha)$, indicates the indirect utility associated with a household with income y in the group α, facing an m-vector of prices.[5] We assume that any two households with the same needs have identical preferences. We also assume V is a continuous and strictly increasing function in income, $\Delta V(p, y, \alpha)/\Delta y > 0$, with the property that $V(p, y, \alpha) < V(p, y, \bar{\alpha}),\ \alpha > \bar{\alpha}$, for all $p \epsilon \mathbb{R}_{++}^{m}$, $y \epsilon \mathbb{R}_{+}$ and $\alpha, \bar{\alpha} \in \Gamma$. It means that

[5] We assume the existence of aggregating function $V(p, y, \alpha)$ that maps individual utilities into household utility. One such possibility is equally distributed equivalent utility (EDE) as in Blackorby and Donaldson (1993). Another one is a weighted average as in the collective choice models (see Browning et al. (2013)). However, for the purpose of the present paper no particular choice has to be made.

the utility associated with households in a group decreases with the levels of needs, that is, $\Delta V(p, y, \alpha)/\Delta\alpha < 0$.[6]

2.2. Equivalence Scales

By inverting $V(p, y, \alpha)$ for each fixed α and p, we obtain the expenditure functions $e(p, u, \alpha)$ which give the minimum cost of utility $u = V(p, y, \alpha)$. Using consumption duality we can derive $y = e(p, u, \alpha)$ where e is the expenditure required to achieve the level of utility u, for a household with needs α, facing prices p. The equivalence scale $ES(p, u, \alpha, \alpha^r)$ is the relative expenditure of household with needs α relative to that of benchmark household with needs α^r (say, single adult), while maintaining the same level of utility u. In other words, once household utility is held constant we can compare income of any household in terms of the income of the reference household.

Assuming all households face the same prices, the equivalence scale is defined:

$$ES(p, u, \alpha, \alpha^r) = \frac{e(p, u, \alpha)}{e(p, u, \alpha^r)} \quad (1)$$

Therefore, the function $ES(p, u, \alpha, \alpha^r)$ can be implicitly defined in terms of the indirect utility function as the value $d = ES(p, u, \alpha, \alpha^r)$ such that:

$$u = V(p, y, \alpha) = V(p, \frac{y}{d}, \alpha^r) \quad (2)$$

[6] We also adopt the following technical assumption as in Blackorby and Donaldson (1991): for all $p \in \mathbb{R}^m_{++}$, $y \in \mathbb{R}_+$, and $\alpha, \bar{\alpha} \in \Gamma$, there exists $\bar{y} \in \mathbb{R}_+$ such that: $V(p, y, \alpha) = V(p, \bar{y}, \bar{\alpha})$. Note that it is unique because V is strictly increasing in y.

Note that the value of $d = ES(p, u, \alpha, \alpha^r)$ in equation (2), is only meaningful up to those monotone transformations of utility for which d does not change. The set of monotone transformations that keep d unchanged characterizes the informational basis, which describes the degree of measurability and comparability of the level of utilities among households.

2.3. Uniform equivalence scale and informational basis

The most common approach in forming welfare comparisons between households is uniform equivalence scales (UES), where $ES(p, u, \alpha, \alpha^r)$ in equation (1) is constant for all levels of u. In this case, UES must satisfy a condition of equivalence scales do not depend on u:

$$UES(p, u, \alpha, \alpha^r) = \frac{e(p,u,\alpha)}{e(p,u,\alpha^r)} = f(p, \alpha, \alpha^r) \quad (3)$$

Such condition is also called Independence of Base (IB) introduced by Lewbel (1989) or Equivalent-Scale Exactness (ESE) proposed by Blackorby and Donaldson (1991,1993).

***Axiom OFC*: Ordinality and full-comparability**[7] (Hammond 1976, d'Aspremont and Gevers (1977, 2002), Roberts (1980a,b), Lewbel 1989, Blackorby and Donaldson (1991, 1993), Fleurbaey 2003, Fleurbaey and Hammond 2004, and Bossert and Weymark 2004): *When making intra- and inter-household comparisons, the following transformations are allowed:*

$$\bar{V}(p, y, \alpha) = \varphi(V(p, y, \alpha)), \qquad \textit{for all } (p, y, \alpha) \textit{ and all } V \quad \textit{(4)}$$

for all $\varphi \in \Phi$, where Φ is the set of all increasing functions.

Note that the intra-household ordinality is satisfied because for all $(p, \bar{p}, y, \bar{y}, \alpha)$:

[7] This condition is implicit in Sen (1974). Hammond (1976) and d'Aspremont and Gevers (1977, 2002) call it "*coordinality*," Roberts (1980a,b) and Fleurbaey (2003) and Fleurbaey and Hammond (2004) call it "*ordinality and level comparability*," Lewbel (1989) calls it "*ordinal level comparability*", and Blackorby and Donaldson (1991, 1993) calls it "*ordinal full-comparability plus.*"

$$V(p,y,\alpha) \geq V(\bar{p},\bar{y},\alpha) \Leftrightarrow \varphi(V(p,y,\alpha)) \geq \varphi\big(V(\bar{p},\bar{y},\alpha)\big),$$

Full inter-household level comparability is also satisfied because for all $(p,\bar{p},y,\bar{y},\alpha,\bar{\alpha})$:[8]

$$V(p,y,\alpha) \geq V(\bar{p},\bar{y},\bar{\alpha}) \Leftrightarrow \varphi(V(p,y,\alpha)) \geq \varphi\big(V(\bar{p},\bar{y},\bar{\alpha})\big),$$

OFC axiom imposes restrictions on the equivalence scales because equivalence scales have to be meaningful (invariant) under the admitted increasing transformations. In particular, Blackorby and Donaldson (1993) noted that OFC is a necessary condition for UES, which we summarize in the following proposition:

Proposition 1: *In order to be meaningful, UES requires OFC axiom. In other words, if* $ES(p,u,\alpha,\alpha^r) = \frac{e(p,u,\alpha)}{e(p,u,\alpha^r)} = f(p,\alpha,\alpha^r)$ *for all* u*, then* u *must be invariant to all* $\varphi(u)$, $\varphi \in \Phi$, *for all* α *and* α^r.

Proof: We need to show that UES ⇒ OFC. Assume $ES(p,u,\alpha,\alpha^r) = \frac{e(p,u,\alpha)}{e(p,u,\alpha^r)} = k$ for some u. Suppose u is not invariant to some $\varphi(u)$ for α and α^r, then the equivalent scale must change:

$$ES(p,u,\alpha,\alpha^r) = \frac{e(p,\varphi(u)\,,\alpha)}{e(p,\varphi(u)\,,\alpha^r)} \neq k$$

which contradicts definition of uniform equivalence scales in (3). QED

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[8]Note that to incorporate inter-household difference comparability, we have to impose a condition such that for all (A,B, $\alpha,\bar{\alpha}$):

$$V(A,\alpha) - V(B,\alpha) \geq V(A,\bar{\alpha}) - V(B,\bar{\alpha}) \Leftrightarrow \bar{V}(A,\alpha) - \bar{V}(B,\alpha) \geq \bar{V}(A,\bar{\alpha}) - \bar{V}(B,\bar{\alpha}).$$

This is a stronger informational framework that requires invariance under common affine transformations.

Note that if we want to keep $ES(p,u,\alpha,\alpha^r)$ unchanged for all utility levels, as standard uniform equivalence scales do, then we need to further restrict the shape of the ordinal utility functions. They must satisfy the IB/ESE condition, for which the expenditure functions can be multiplicatively written as, for all (p,u,α), (Lewbel, 1989):

$$e(p,u,\alpha) = e_1(p,u)e_2(p,\alpha) \quad (5)$$

2.4. Informational basis and general equivalence scale

Even though uniform equivalence scale is a convenient simplification in order to make inter-household comparisons of welfare, its validity is rejected empirically for at least some comparisons (Pendakur, 1999). Therefore, a more general approach is desirable where $ES(p,u,\alpha,\alpha^r)$ in equation (1) can vary for different levels of u. We shall call this general equivalence scales (GES):

$$GES(p,u,\alpha,\alpha^r) = \frac{e(p,u,\alpha)}{e(p,u,\alpha^r)} = h(p,u,\alpha,\alpha^r) \quad (6)$$

Intuitively it would allow poor and rich households to require very different relative compensations when the demographic structure changes. To best illustrate it imagine newborn in household living on subsistence level versus those who are non-poor. Even though the difference in absolute cost may not be large, the proportional increase on its initial income for low-income household should be much higher.

When considering a general equivalence scale one needs to specify informational basis in order to determine admissible utility functions and their properties. Preferably we want to have only the restrictions imposed by the Arrovian *ordinal non-comparability* case (ONC).

***Axiom ONC*: Ordinality and non-comparability** (Arrow, 1963): *When making intra- and inter-household comparisons, the following transformations are allowed:*

$$\hat{V}(p, y, \alpha) = \varphi_\alpha(V(p, y, \alpha)), \quad \textit{for all } (p, y, \alpha) \textit{ and } V \qquad (7)$$

for all $\varphi_\alpha \in \Phi$.

Note that no inter-household comparisons are allowed. Even though it is desirable to construct equivalence scales satisfying only ONC, it is not possible to do because by definition some degree of comparability is required.

Thus, one needs to consider more restrictive case than ONC but less than OFC. In the next section we show the least restrictive informational basis guaranteeing meaningful GES.

2.5. Minimum requirements for informative general equivalence scale

The least restrictive case for informative equivalence scales must allow at least ordinal utility framework (utility is equivalent up to any monotone transformation within the households) and the ability for some comparability among households (inter-households comparability restrictions must be specified). These conditions provide enough information to specify new informational basis. Thus, we propose new axiom:

***Axiom OLC*: Ordinal local-comparability axiom**. *Assume a common value of utility* $\bar{u}$ *is identified for all households. The following transformations* $\tilde{V}$ *are allowed to make intra- and inter-household comparisons if for all* α, p *and* y, $u = V$ *is replaced by* $\tilde{V}$:

$$\tilde{V}(p, y, \alpha) = \varphi\big(V(p, y, \alpha)\big), \quad \text{for all } (p, y, \alpha) \text{ and } V = \bar{u} \qquad (9a)$$

and

$$\tilde{V}(p,y,\alpha) = \varphi_\alpha(V(p,y,\alpha)), \quad \text{for all } (p,y,\alpha) \text{ and } V \neq \bar{u} \tag{9b}$$

where φ, $\varphi_\alpha \in \Phi$.

The underlying informational basis is very general and supports ordinal and local inter-household comparability of utilities at a particular utility level. In this case intra-household ordinality is satisfied because for the same household and for all $(p, \bar{p}, y, \bar{y}, \alpha)$:

$$V(p,y,\alpha) = V(\bar{p},\bar{y},\alpha) \Leftrightarrow \varphi\big(V(p,y,\alpha)\big) = \varphi\big(V(\bar{p},\bar{y},\alpha)\big), \qquad V = \bar{u}$$

$$V(p,y,\alpha) \geq V(\bar{p},\bar{y},\alpha) \Leftrightarrow \varphi_\alpha(V(p,y,\alpha)) \geq \varphi_\alpha\big(V(\bar{p},\bar{y},\alpha)\big), \qquad V \neq \bar{u}$$

Moreover, when comparing different households, inter-household level comparability is satisfied at $V = \bar{u}$ because for all $(p, \bar{p}, y, \bar{y}, \alpha, \bar{\alpha})$:[9]

$$V(p,y,\alpha) = V(\bar{p},\bar{y},\bar{\alpha}) \Leftrightarrow \varphi\big(V(p,y,\alpha)\big) = \varphi\big(V(\bar{p},\bar{y},\bar{\alpha})\big), \qquad V = \bar{u}$$

but it is not satisfied at $V \neq \bar{u}$ because for all $(p, \bar{p}, y, \bar{y}, \alpha, \bar{\alpha})$:

$$V(p,y,\alpha) \geq V(\bar{p},\bar{y},\bar{\alpha}) \nRightarrow \varphi_\alpha(V(p,y,\alpha)) \geq \varphi_{\bar{\alpha}}\big(V(\bar{p},\bar{y},\bar{\alpha})\big), \qquad V \neq \bar{u}$$

and

$$V(p,y,\alpha) \geq V(\bar{p},\bar{y},\bar{\alpha}) \nLeftarrow \varphi_\alpha(V(p,y,\alpha)) \geq \varphi_{\bar{\alpha}}\big(V(\bar{p},\bar{y},\bar{\alpha})\big), \qquad V \neq \bar{u}.$$

Note that the class of monotone transformations allowed under OLC is wider than OFC, and therefore the criterion requires much less information typically assumed in the literature for uniform equivalence scales.

[9] The reader will note that under this specification full inter-household level comparability is also satisfied for households belonging to the same group. Less demanding informational basis is still possible by admitting individual specific monotone transformations.

Alternatively, we can express IB and OLC conditions in terms of the indirect utility functions (the income-ratio comparability condition by Blackorby and Donaldson, 1991), for all $(p, \bar{y}, \tilde{y}, \bar{\alpha}, \tilde{\alpha})$:

$$V(p, \bar{y}, \bar{\alpha}) = V(p, \tilde{y}, \tilde{\alpha}) \Leftrightarrow V(p, \lambda\bar{y}, \bar{\alpha}) = V(p, \lambda\tilde{y}, \tilde{\alpha}) \qquad \text{for} \quad \lambda > 0$$

for all V for the IB case and only for $V = \bar{u}$ under the OLC case.

2.6. Equivalence scale under OLC

OLC framework certainly has implications for the equivalence scales. In the OLC case, the expenditure function is less restrictive. It can be written as:[10]

$$e(p, u, \alpha) = H[e(p, u, \alpha)] \tag{10}$$

where $H[e(p, u, \alpha)] = \begin{cases} e(p, \bar{u}, \alpha), u = \bar{u} \\ e(p, u, \alpha), u \neq \bar{u} \end{cases}$ (11)

Note that the equivalence scale $ES(p, \bar{u}, \alpha, \alpha^r)$ is:

$$ES(p, \bar{u}, \alpha, \alpha^r) = \frac{e(p,\bar{u},\alpha)}{e(p,\bar{u},\alpha^r)} \tag{12}$$

However ES derived from expenditure functions in equation (10) may depend on u for $u \neq \bar{u}$. We define the local equivalence scales (LES) as the ones that are constant for u=$\bar{u}$. In this case, LES must satisfy equivalence scales do not depend on u for $u = \bar{u}$:

$$LES(p, \bar{u}, \alpha, \alpha^r) = \frac{e(p,\bar{u},\alpha)}{e(p,\bar{u},\alpha^r)} = f(p, \bar{u}, \alpha, \alpha^r) \quad \text{for } u = \bar{u} \tag{13}$$

We prove that to be meaningful, local equivalence scales must satisfy OLC.

[10] This formulation opens the gate to expand the methodology to multiple-level comparability. Assume we can identify two comparable utility levels, say $\bar{u}$ and $\bar{\bar{u}}$ then we could write the ordinal multi-level OMLC as: $H[e(p, u, \alpha)] = \begin{cases} e(p, \bar{u}, \alpha), u = \bar{u} \\ e(p, \bar{\bar{u}}, \alpha), u = \bar{\bar{u}} \\ e(p, u, \alpha), u \neq \bar{u} \neq \bar{\bar{u}} \end{cases}$
Note that when multi-level goes to infinity we converge to GES.

Proposition 2: *Any local equivalence scale (LES) as defined by (13) and with fixed $u = \bar{u}$ is consistent with OLC axiom. In other words, if $LES(p, \bar{u}, \alpha, \alpha^r) = \frac{e(p,\bar{u},\alpha)}{e(p,\bar{u},\alpha^r)} = f(p, \bar{u}, \alpha, \alpha^r)$ for $u = \bar{u}$, then u must be invariant to all $\varphi(u)$, $\varphi \in \Phi$, for all α and α^r for $u = \bar{u}$.*

Proof*:* We need to show that LES ⇒ OLC. Assume $LES(p, \bar{u}, \alpha, \alpha^r) = \frac{e(p,\bar{u},\alpha)}{e(p,\bar{u},\alpha^r)} = k$ for some $u = \bar{u}$. Suppose u is not invariant, then there exists some $\varphi(u)$ for α and α^rsuch that the equivalent scale must change:

$$ES(p, \bar{u}, \alpha, \alpha^r) = \frac{e(p, \varphi(\bar{u}), \alpha)}{e(p, \varphi(\bar{u}), \alpha^r)} \neq k$$

which contradicts definition of local equivalence scales in (13). QED

2.7. Practical solution

Under our framework for a single equivalence scale which satisfies OLC, we opt for a practical solution suggested by Blackborby and Donaldson (1993) which can be very relevant for policy analysis and for identification of the demand system as in Blundell and Lewbel (1991). We compare utility levels at a particular level where it is required that households to "make ends meet," that is, when $\bar{u} = u_{min}$. Then we can write the "minimum needs" equivalence scale as:

$$MES(p, u_{min}, \alpha) = \frac{e(p, u_{min}, \alpha)}{e(p, u_{min}, \alpha^r)} \tag{14}$$

We assume that at the poverty line all households have the same comparable utility, for some fixed prices.

Assumption 1: *Poverty equivalent utility.*

At the poverty line $y^*_{min}(\alpha)$, all households attain the same utility. For all p and $\alpha \in \Gamma$:

$$u_{min} = V(p, y^*_{min}(\alpha), \alpha)$$

Note that the benefit of adopting assumption 1 in the context of OLC is that we can write equivalent scales in equation (14) as:

$$MES(p, u_{min}, \alpha) = \frac{e(p,u_{min},\alpha)}{e(p,u_{min},\alpha^r)} = \frac{y^*_{min}(p,\alpha)}{y^*_{min}(p,\alpha^r)} \tag{15}$$

Specification in (15) turns out very useful because it may help with the identification problem common in estimation of the demand system. Blundell and Lewbel (1991) showed that with the use of the demand data we can at the most estimate preferences over goods conditioned on households characteristics, but we cannot infer anything about the impact of changes of the characteristics on the preferences themselves. In other words, demand system estimation does not capture an important aspect of the demographic characteristic costs. However, Blundell and Lewbel (page 54, 1991) showed that we can decompose:

$$MES(p, u_{min}, \alpha) = \frac{e(p,u_{min},\alpha)}{e(p,u_{min},\alpha^r)} = \left(\frac{e(p,u_{min},\alpha)/e(p^r,u_{min},\alpha)}{e(p,u_{min},\alpha^r)/e(p^r,u_{min},\alpha^r)}\right)\left(\frac{e(p^r,u_{min},\alpha)}{e(p^r,u_{min},\alpha^r)}\right) \tag{16}$$

It follows from this equation (16) that the equivalence scale in price regime *p* equals the product of two terms in the left-hand side of the equation (16). The first term is a ratio of household-specific cost of living indices, which can be identified from demand data alone, and, the second term is the equivalence scale at the price reference regime p^r, which cannot be estimated empirically unless we observe the impact of changes in characteristics. Notice that the second component is equivalent to equation (15), and therefore once it is available then it significantly reduces the identification problem. In fact, Blundell and Lewbel (1991) suggested using subjective data to obtain values for (15) (see subsection (iii) on p. 57), and our paper can be viewed as a demonstration that such approach is justified.

We devote the rest of the paper to identify $y^*_{min}(p, \alpha)$ for all $\alpha \in \Gamma$ using subjective data. We formulate the intersection method proposed by Goedhart *et al.* (1977), using minimum needs income subjective data.

3. MINQ-BASED DATA AND THE POVERTY LINE: THE INTERSECTION METHOD

The derivation of the poverty line using subjective questions was first proposed by Goedhart et al. (1977). They introduce two approaches: *Leyden Poverty Line* (LPL) based on multi-level question, and *Subjective Poverty Line* (SPL) based on a one-level question. Even though Flik and Van Praag (1991) argue that LPL is theoretically superior to the SPL[11], we suggest that SPL may be preferred because it is less restrictive and can easily be integrated with any other study which requires specification of the indirect utility function of income.

3.1. Subjective Poverty Line

The SPL is based on the answer to the minimum needs question (MINQ): "what is the minimum income that you would have to have to make ends meet?" Even though for a given group of households the answers to MINQ vary due to "misperception," Goedhart et al. (1977) argued that there is a systematic relationship between answers to MINQ and an actual income[12]. In particular, those with actual income above their minimum income overestimate the poverty

[11] Many question theoretical validity of LPL, see for example Seidl (1994). For a discussion regarding SPL see Ravallion (2012).

[12] In the discussion of subjective utility evaluations Roberts (1997) suggests that households may make “mistakes” when comparing themselves to those that are different from them, say poor vs. non-poor, but their opinion may correctly reflect objective comparisons when they compare themselves to households which are similar, say all households that have just enough income to meet their needs.

threshold, whereas those with actual income below their minimum income underestimate the poverty threshold. We further call it perception error.

According to Goedhart et al. (1977) only those whose actual income equals minimum needs income would answer MINQ correctly. The intuition is that at this level of income the household has neither savings nor incurs additional debt, and any shortfall would push it into poverty. Therefore it becomes a definition of the poverty line for a particular group of households with the same characteristics.

Unfortunately because most samples do not include households whose actual income equals the answer to MINQ, or their numbers in the population may be very limited, it is not possible to directly use answers to MINQ in the formulation of the subjective poverty line. However, Goedhart et al. (1977) shows that by modeling the perception error in MINQ one can use data from the entire survey to estimate the correct answer to MINQ (true minimum needs income). The objective of their approach is to find such actual income level where households would not make a perception error when reporting MINQ. The technique is called the *intersection method* because graphically the poverty line is at the point of intersection between the line representing survey answers to MINQ conditional on actual income, and the line representing hypothetical answers to MINQ as if the answers would always equal to the actual income, the 45 degree line (see figure 1). A formal presentation of the intersection method follows.

[Figure 1 about here]

3.2. Intersection method

Suppose that for all households in the subpopulation with α characteristics, there exists unique true minimum needs income (poverty line) which is unobservable, defined as $y^{*}_{min}(\alpha)$.

What is observable are the answers to MINQ which are "distorted by the fact that (respondent's) actual income is not equal to his minimum income" (Goedhard et al. 1977, p. 514). Thus, to represent actual answers to MINQ we define "minimum needs income" perception function (IPF) of subpopulation α, $f_p(y,\alpha)$, f_p: $[0, y_{max}(\alpha)] \rightarrow [0, y_{max}(\alpha)]$, where $y_{max}(\alpha)$ is the observed maximum income of the group α; which depends on actual income y, assumed to be continuous in y with $\frac{\Delta f_p(y,\alpha),}{\Delta y} > 0$. The difference between IPF and $y^*_{min}(\alpha)$ is the perception error which is systematic, such that poor households have $f_p(y,\alpha) - y^*_{min}(\alpha) < 0$, and non-poor households have $f_p(y,\alpha) - y^*_{min}(\alpha) > 0$.

It has to be noted that the perception error is different from random error, which is present when estimating IPF using survey data. If we denote $y_{min}(\alpha)$ as the reported answer to MINQ in survey data, the random error is the difference: $y_{min}(\alpha) - f_p(y,\alpha)$. Therefore, the full decomposition of the errors becomes (see Figure 1):

$$y_{min}(\alpha) - y^*_{min}(\alpha) \quad = \quad \left[f_p(y,\alpha) - y^*_{min}(\alpha)\right] \quad + \quad \left[y_{min}(\alpha) - f_p(y,\alpha)\right] \qquad (17)$$

total reported error = perception (systematic) error + random error

Throughout the paper we ignore random error (assume it is equal to zero) because it is data-specific and the derivation of SPL does not depend on the properties of the random error. In other words, modeling random error is the subject of econometric specification, which is not the focus of the present paper.

The objective of the intersection method is to use answers to MINQ in order to find the unobserved minimum needs income, $y^*_{min}(\alpha)$. The ingenuity of the approach is that we can find such income even if there are no households in the data for whom $y = y^*_{min}(\alpha)$ (otherwise the solution if obvious). Following the specification in Kapteyn, Kooreman, and Willemse (1988), the method is based on the existence of IPF such that $y^*_{min}(\alpha)$ is the unique solution to:[13]

$$y^*_{min}(\alpha) = f_p(y^*_{min}(\alpha), \alpha) \tag{18}$$

where for $y < y^*_{min}(\alpha)$ we have that $y < y_{min}(\alpha)$ and for $y > y^*_{min}(\alpha)$ we have that $y > y_{min}(\alpha)$. Because there is no random error it means that for $y > y^*_{min}(\alpha)$ we have $y < f_p(y, \alpha)$ and for $y > y^*_{min}(\alpha)$ we have $y > f_p(y, \alpha)$.

Solution to (18) is presented in Figure 1, where vertical axis represents values of $f_p(y, \alpha)$ (assumed linear for demonstration purposes) conditional on actual income y (horizontal axis). The 45 degree line includes all the points where the condition $y = y^*_{min}(\alpha)$ is satisfied. Therefore intersection of 45 degree line with the function $f_p(y, \alpha)$ is the solution to the problem in (18).

In practice, because there is random error one needs econometric model to estimate IPF. However, the unique feature of SPL is that the functional form of IPF is irrelevant as long as it is monotonically increasing in both y, and its distance from actual income is always smaller than the distance between actual income and true minimum needs income, $\left|y - f_p(y, \alpha)\right| < |y - y^*_{min}(\alpha)|$.[14]

3.3. Formalization of the intersection method

In the following we propose a formalization to justify the existence of the SPL implied by the methods originally presented in Goedhart et al. (1977). The focus is on demonstrating that those methods do not restrict the underlying utility function to any particular functional form and

[13] This unique solution is implied by Assumption 2 below. See Proposition 3 for formal proof.
[14] We want to note that it is a very important distinction between SPL and LPL because the functional form of IPF in LPL is the double-log specification, which is derived from the assumed functional form of WFI.

thus ordinal utility specification is acceptable. In other words, the intersection method is consistent with OLC, defined in section 2.5. OLC is the only comparability axiom required for MINQ-based method to derive the SPL and the corresponding subjective equivalence scales. However, OLC is not unique to MINQ-based subjective equivalence scale because there may be other methods which allow identification of households with the same level of utility.

We argue that two assumptions are needed for SPL. The first assumption was introduced in section 2.7 and is based on single-level equivalence scale proposed by Blackboard and Donaldson (1993). The second assumption is technical and deals with the properties of the IPF.

Assumption 2: ***Minimum Income Perception.*** $\forall y \in \mathbb{R}_+$ and $\forall \alpha \in \Gamma$:

$$y > y^*_{min}(\alpha) \Rightarrow y > f_p(y,\alpha) > y^*_{min}(\alpha) \quad (19a)$$

and

$$y < y^*_{min}(\alpha) \Rightarrow y < f_p(y,\alpha) < y^*_{min}(\alpha) \quad (19b)$$

(The $\Leftarrow$ is obvious and is omitted). It says that those households with actual income below their minimum income underestimate the poverty threshold, whereas those with actual income above their minimum income overestimate the poverty threshold.

The intuition is that poor households at the very least must be aware that they are poor ($y \leq f_p(y,\alpha)$ and they always underestimate the degree of their poverty ($f_p(y,\alpha) \leq y^*_{min}(\alpha)$).

The interpretation of assumption 2 is that even though IPF can take very flexible functional forms, it has to be restricted to guarantee unique solution to (18). In other words, there cannot be multiple intersection points. We formalize it in the following proposition:

Proposition 3: *Consider a continuous perception function on a convex compact set satisfying assumption 2, then the unique solution $y_0 = f_p(y_0, \alpha) = y^*_{min}(\alpha)$ exists.*

Proof: Define the perception functionf_p: $[0, y_{max}(\alpha)] \rightarrow [0, y_{max}(\alpha)]$ as a continuous mapping of a convex compact set in itself. Brouwer's fixed-point theorem guarantees that there exist at least one fixed-point y_0 for all α such that $f_p(y_0, \alpha) = y_0$. Then $y_0 = f_p(y_0, \alpha) = y^*_{min}(\alpha)$.

We prove uniqueness by considering a direct implication of assumption 2. For any particular income value $y \in \mathbb{R}_+$, one of the following expressions is correct:

$$y > f_p(y, \alpha) > y^*_{min}(\alpha), \tag{20a}$$

$$y = f_p(y, \alpha) = y^*_{min}(\alpha) \tag{20b}$$

$$y < f_p(y, \alpha) < y^*_{min}(\alpha) \tag{20c}$$

QED.

The intuition of proposition 3 is that the intersection method will always produce a unique level of income for poor households as long as we can correctly model the perception function. It implies that under this condition we can identify the poverty line $y^*_{min}(\alpha)$ for every needs group α, from subjective MINQ-based data.

Finally, we can obtain the underlying equivalence scales by substituting the $y^*_{min}(\alpha)$ values in equation (15). Recall that by assumption 1, at the poverty lines, two households with different needs α and $\bar{\alpha}$ attain the same (ordinal and comparable) utility:

$$V(p, y^*_{min}(\alpha), \alpha) = V(p, y^*_{min}(\bar{\alpha}), \bar{\alpha}) \tag{21}$$

Note that if $\alpha < \bar{\alpha}$ then $y^*_{min}(\alpha) < y^*_{min}(\bar{\alpha})$ due to $\Delta V/\Delta y > 0$ and $\Delta V/\Delta\alpha < 0$. Thus, as one would expect, the intersection method imposes higher poverty lines for households with higher needs. The equivalence scale for group $\bar{\alpha}$ with respect to group α at the poverty threshold is just $MES = y^*_{min}(\bar{\alpha})/y^*_{min}(\alpha)$. See Figure 2 for an illustration.

3.4. OLC and assumptions of the MINQ-based subjective equivalence scales

We can now demonstrate that MINQ-based subjective equivalence scale obtained using the intersection method requires OLC. We need to show that the assumptions underlying the subjective scale requires OLC.

We begin by reiterating the intersection method is just one possibility of estimating MES scale introduced in section 2.7, which compares only households living at the poverty level. In addition to assumption (1) it requires assumption (2). However, the latter is technical in nature and does not involve any restrictions, outside the poverty line, on the underlying utility function. Therefore, because MES scale is an example of LES scale, we can summarize our result in the following Corollary to Proposition 2:

Corollary: *Any MINQ-based subjective equivalence scale derived using the intersection method is consistent with OLC axiom. In other words, if* $MES(p, u_{min}, \alpha, \alpha^r) = \frac{e(p,u_{min},\alpha)}{e(p,u_{min},\alpha^r)} = f(p, u_{min}, \alpha, \alpha^r)$ *for u=* u_{min}*, then* u *must be invariant to all* $\varphi(u)$*,* $\varphi \in \Phi$*, for all* α *and* α^r *for* $u = u_{min}$.

The proof is equivalent to derivations presented for proving Proposition 2 and is omitted.

The result means that any MINQ-based subjective equivalence scale has very minimal requirements regarding the underlying utility function. It is in stark contrast to uniform equivalence scale implying IB/ESE preferences, a condition rejected empirically for some comparisons (Pendakur, 1999). Therefore, our paper presents an alternative approach to equivalence scales calculation which allows a relaxation of those conditions.

It needs to be noted that Corollary does not say anything about validity or quality of the available subjective data as a reflection of true preferences for households. It also does not provide an insight into an implementation of the intersection method as a way to calculating the minimum needs income. This is because the perception error is separate from the stochastic error term in the empirical econometric model, which is ignored in this paper. We acknowledge that addressing those issues may present significant challenges and must be confronted in future research. However, given that issues of implementation are overcome and the empirical work is property done, Corollary demonstrates that MINQ-based subjective equivalence scales give a possibility to obtain true GES at any selected utility level.

3. CONCLUSION

Uniform equivalence scales, which assume the same adjustment of incomes across households with different utility levels, are a convenient practical simplification used in empirical studies. However, such an approach imposes the IB/ESE condition which requires Ordinal Full Comparability (OFC) of utility profiles. In turn, it limits the possibilities of preferences that can be used. We demonstrate that an approach based on using a single utility level to calculate the equivalence scale requires much a weaker condition. In particular, we focus only on households living at the poverty line income. Such an approach only needs to satisfy the axiom of Ordinal and Local Comparability (OLC) that allows for non-comparable preferences except for at a single preference level.

One concern with using an equivalence scale calculated for a single subgroup is that it ignores information about the rest of the population. In other words, one may argue that such an approach does not allow for the generalization required to perform income comparisons across households with different utility levels. However, if the preferences satisfy IB/ESE then a single equivalence scale is automatically correct for the entire population. This conclusion relates to the fact that a preference ordering satisfying OFC also satisfies OLC. The benefit of OLC, however, is that it does not require particular IB/ESE preferences.

Even if one would reject IB/ESE and argue that there are different equivalence scales at each level of utility, there is still a benefit of using the minimum needs income approach because often times welfare policy focuses on that particular subgroup of the population. Therefore, the information about the most efficient way to improve the situation of the households at the bottom of the income distribution is particularly useful, even if it ignores the rest of the population.

An additional benefit of minimum needs income equivalence scales is the ability to construct a test for IB/ESE. Similarly to the survey available to Geodhart et al. (1977), one could ask multiple subjective questions regarding levels of incomes for households to "meet their minimum needs," "meet their needs to be middle class," or "meet their needs so that they regard themselves wealthy," etc. The answer to each question provides information about a unique utility level. With the use of the intersection method it should be possible to calculate multiple equivalence scales and test whether they are the same or different.

Finally, Blundell and Lewbel (1989) showed the limits of information provided by the demand data alone for the purpose of deriving equivalence scales, and they suggested that availability of a correct equivalence scale at the selected reference utility level can help in identification (see lemma on p. 52). Thus, the minimum needs income approach, which satisfies

OLC, can provide important identifying information which is inherently lacking when estimating equivalence scales using a demand system. Note that MINQ-based equivalence scale can be easily integrated because of its minimal requirements imposed on the utility function.

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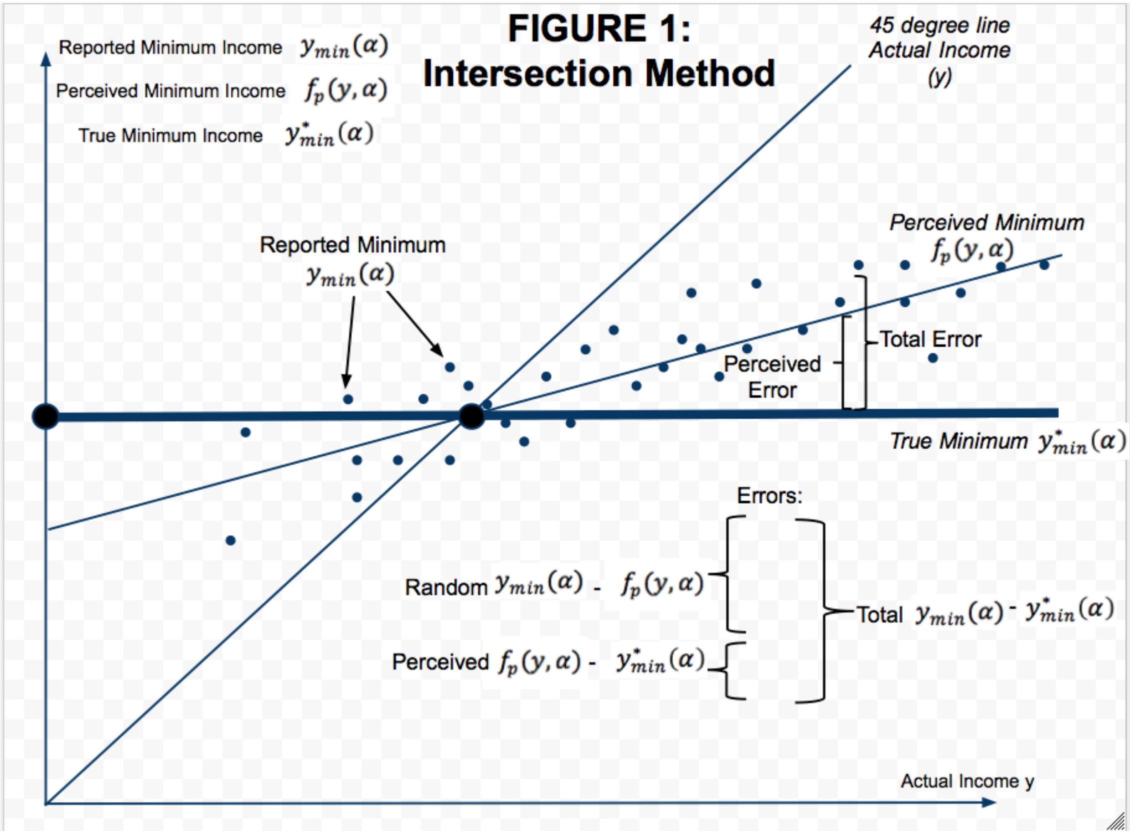
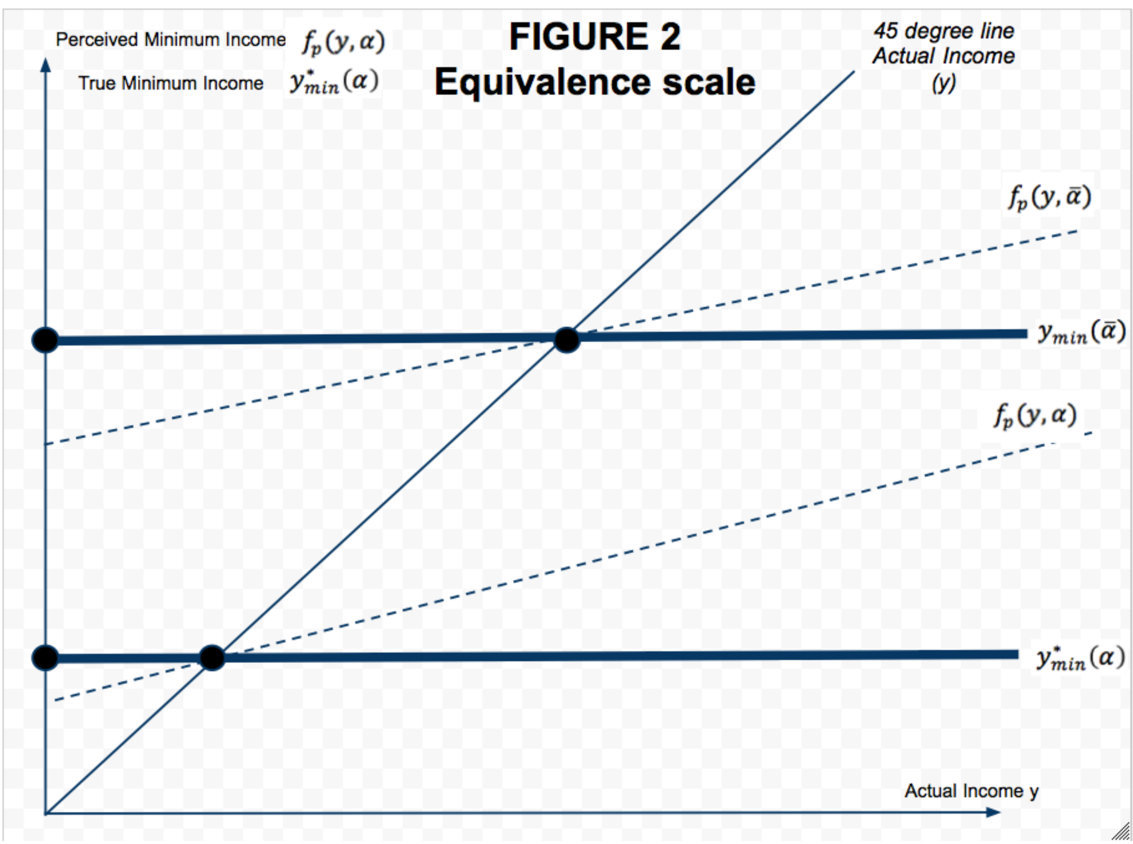


Reported Minimum Income $y_{min}(\alpha)$
Perceived Minimum Income $f_p(y,\alpha)$
True Minimum Income $y^*_{min}(\alpha)$
FIGURE 1:
Intersection Method
45 degree line
Actual Income
(y)
Perceived Minimum
$f_p(y,\alpha)$
Reported Minimum
$y_{min}(\alpha)$
Total Error
Perceived
Error
True Minimum $y^*_{min}(\alpha)$
Errors:
Random $y_{min}(\alpha)$ - $f_p(y,\alpha)$
Total $y_{min}(\alpha)$ - $y^*_{min}(\alpha)$
Perceived $f_p(y,\alpha)$ - $y^*_{min}(\alpha)$
Actual Income y



Perceived Minimum Income $f_p(y,\alpha)$
True Minimum Income $y^*_{min}(\alpha)$
FIGURE 2
Equivalence scale
45 degree line
Actual Income
(y)
$f_p(y,\bar{\alpha})$
$y_{min}(\bar{\alpha})$
$f_p(y,\alpha)$
$y^*_{min}(\alpha)$
Actual Income y