EQUALITAS

Economics of Inequality and Poverty Analysis

EQUALITAS Working Paper No. 64

Intergenerational Mobility and Reference Groups

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September, 2019

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September 26, 2019

Abstract

We model the role of reference groups as a mechanism for inequality persistence across generations. Reference group theory suggests that culturally shaped processes alter individuals' ambition. As a result, relative deprivation effects may discourage (encourage) low-background individuals from making adequate mobility-enhancing investments. The model confirms that reference groups could be an inequality transmission mechanism across generations, and shows that both the size and direction of this effect depends on, (a) the composition of the reference group, (b) the intensity and functional form of income comparisons, (c) the ex-ante inequality between agents from different social origins and the reward of effort, and (d) the information about their peers and past income mobility. Our model is more general than previous models and its findings are in stark contrast to models based upon self-fulfilling beliefs and fatalistic predictions. Finally, our model explicitly links two strands of the literature: Reference group theory and aspiration failure models.

Keywords: Reference group, prospect theory, intergenerational mobility, relative deprivation, effort, Bayesian learning.
JEL codes: A13, D63, J62, Z13.

*We would like to thank Maarten Vendrik, Patricio Dalton, Xavier Raurich, Dirk Van de gaer, Robert Frank and Verónica Frisancho for their comments and suggestions as well as participants at the 6th ECINEQ Meeting in Luxembourg and at the Workshop on Inequality, Poverty and Politics, RIDGE 2016. Xavier Ramos acknowledges financial support of projects ECO2013-46516-C4-1-R (Ministerio de Ciencia y Tecnología), ECO2016-76506-C4-4-R, and SGR2017-1571 (Generalitat de Catalunya). Martín Leites acknowledges financial support from Fundación Carolina.

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1 Introduction

Reference group theory (Boudon, 1974) suggests that culturally shaped processes may affect economic expectations, ambition or taste for economic success of individuals. As a result, social origin and reference groups may shape individual's mobility expectations, economic aspirations or taste for effort, thus becoming a mechanism for the transmission of economic advantage across generations (Piketty, 2000).

Recent evidence for the US, for instance, shows that the environment of places matters for intergenerational mobility (Chetty and Hendren, 2018a). Exposure to favourable socioeconomic characteristics of places, such as low concentration of poverty, reduced inequality and criminality, and better schools, produce better outcomes for children in poor families (Chetty and Hendren, 2018b). In moving to better places, individuals are exposed to better environments which change individuals' reference group. Of course, the environment of neighbourghoods is but one of the many factors that explain how reference groups are chosen.

This paper explores how individuals' economic opportunities are shaped by reference groups. In its simplest form, the idea is that poor individuals who only know and mingle with people of their same condition, see their ambition, taste for effort, or information about feasible opportunities constrained by their social environment. However, similarly poor individuals who are (also) exposed to higher income individuals with better life conditions and different social norms can decide whether to increase their effort and catch up with them or not to and give up the possibility to climb up the social ladder, possibly because inequality seems irreversible to them or because they prefer avoiding frustration. **Introduce a quote from Poverty Safari here**.

Our approach incorporates the idea that agents' objective function considers the self-perceived valuation of their relative position in their reference group.[1] The composition of reference groups defines a reference income level, and agents care about the gap between their income and their reference income. We model rational agents from two different social origins who choose the level of effort that maximizes their expected utility, assuming that they know the relative importance of effort and of predetermined factors for the achievement of economic success. We do this exercise under two different assumptions about the utility function. On the one hand, we assume diminishing marginal utility from standard neoclassical theory. On the other, we follow the tenets of prospect theory and assume loss aversion and diminishing sensitivity.[2] This way we accommodate recent

[1] Empirical evidence suggests the relevance of relative concern regarding human motivations (Frank, 2005), worker's effort decisions (Huet-Vaughn, 2015), and economic satisfaction (Card et al., 2012).

[2] Loss aversion refers to the tendency of people to give more importance to losses relative to a reference

developments in behavioural economics that show how "pervasive and fundamental is the role of changes [...] in assessing the behavior and welfare of individuals" (Rabin, 2002). To gain a first insight, we first consider a simple scenario where individuals know the composition of their reference group and also have perfect information about their reference group income (forward-looking agents). In a second exercise, we assume that agents have imperfect information about the expected effort of their reference group and base their choices on *a priori* beliefs about the probability of economic success of different social origins. Beliefs are updated according to Bayes' rule, implying that past mobility affects the expected income of the current generation. This framework allows us to derive long term effort equilibrium levels and to examine the effect of relative concern on the dynamics of intergenerational mobility.

Our results characterize the situations where relative concerns induce individuals to put in more effort, and thus where upward mobility is more likely. When agents are assumed forward-looking (i.e. expected effort is known), two elements govern effort decisions: the (dis)utility of effort and the gains in utility of lower relative deprivation. Two factors determine the latter, the parameters that determine the probability of economic success, such as returns to effort and ability or ex-ante inequality, and individuals' response to the economic outcomes, i.e. the shape of their utility function. When effort is seen as a cost, individuals will put in more effort whenever this cost is outweighed by the gains from lower deprivation. When agents derive utility from effort, effort always pays.

When expected effort is unknown and effort beliefs are based on the mobility of the previous generation, the results that emerge from our model confirm that reference groups affect inequality of economic success between individuals from different social origins, because of the relative income effect and aspirations conformation. The assumed functional form of relative concern, the composition of reference groups and past mobility trajectories for agents from different social origins easily generate multiple equilibrium in effort levels. That is, effort levels of agents with identical ability differ in the long term, which affect long-term income mobility, and also persistent inequality. Our results suggest that the size and direction of the relative income effect depend on four key issues: (a) the composition of the reference group, which is relevant regardless of inheritance patterns between generations; (b) assumptions about the functional form of relative concern; income mobility is very different when the functional form of relative concern accords with standard assumptions or prospect theory assumptions; (c) ex-ante inequality and relative effort rewards; (d) expected effort beliefs and past mobility perceptions.

As in previous models where comparisons matter for individual utility (Clark and Oswald, 1998; Piketty, 1998; Frank, 1997; 2005), our model yields a suboptimal equilibrium because agents ignore the externalities of their effort decisions. We identify two sources of

point than to gains. Diminishing sensitivity is the tendency of people to put less weight on marginal changes for changes that are further away from the reference point.

externalities. A first one happens between individuals from different social origin, as the effort of higher origin agents generates a negative externality on the decisions of agents from lower origin, the inefficiency being larger the higher the inequality between agents from different social backgrounds. The second source of externality comes from within agents of the same lower origin.

Our model helps understand recent stylized facts in the literature regarding inequality and mobility, such as the so-called Great Gatsby curve, which shows the positive relationship between income inequality and intergenerational mobility (Solon, 2002; Krueger, 2012; Corak, 2013; Mitnik et al., 2015; Chetty et al., 2014). It also allows to interpret the evidence about heterogeneous aspirations and adaptive preferences hypothesis (Festinger, 1975; Sen 1985a; 1985b; Elster, 1985; Clark, 2009). Furthermore, it also helps explain situations of low mobility for certain social groups and contributes to explain why agents with a similar family background and abilities have different economic success.

We contribute to the theoretical literature on the socio-cultural mechanisms of inequality persistence, in particular to the literature that models the influence of reference groups (Postlewaite, 1998; Weiss and Ferschtman 1998; Austen-Smith and Fryer, 2005). We build on Piketty (1998), that models how public beliefs about one's ability (social status) and self-fulfilling beliefs may lead to persistent inequality. Piketty views his theory of persistent inequality through the status motive as very similar to Bourdieu's sociological theory, which suggests that the way the dominant discourse in capitalist societies discourages lower-class individuals from seeking to socially progress and encourages instead to settle for less prestigious social outcomes is largely responsible for persistent inequality (Bourdieu and Passeron, 1964 and 1970). Piketty also relates his theory with the reference group theory (Boudon, 1974). However, unlike our model, Piketty's model does not accommodate two important features that derive from reference group theory, namely, people care about their relative position with respect to a reference point, and people react differently depending on the composition of their reference group. Furthermore, as in Piketty (1998) we first assume forward-looking agents, but unlike Piketty (1998) we also consider the situation where agents do not know the effort of their peers and update their effort beliefs by a backward-looking learning process. Finally, even though we also model two social origins, we allow individuals from the same social origin to have different effort equilibrium levels depending on their reference group, introducing thus heterogeneity within social class. The introduction of these four features in our model allows us to examine the conditions under which, and the way, reference groups affect income mobility in a way that is not possible in Piketty's model. Our model is more general than previous models and its findings are in stark contrast to models based upon self-fulfilling beliefs and fatalistic predictions, when relative income and leisure are assumed complements.

Our paper speaks to the recent literature on aspirations (Appadurai, 2004; Ray, 2006; Genicot and Ray, 2017). First, if we assume that income aspiration is the income of the reference group, our paper provides a framework to discuss the conditions that lead to the two types of aspiration failure identified by Ray (2006) –when agents from low social origin do not include agents from high social origin in their aspiration window, and when previous inequality and the relative costs of effort are so high that agents perceive the goal to be unattainable and are thus discouraged. Our paper differs from Dalton et al. (2016) in that we focus on the external or social conditions whereas they study how a behavioural bias (an internal constraint) influences the formation of aspirations differently for poor and non-poor individuals.

This complementarity also provides new arguments to the policy discussion. Piketty (2000) argues that sociocultural inequalities could generate extra inequality persistence, where intergenerational mobility would be inefficiently low. In this context, appropriate corrective policies (or alternative wealth distribution) could raise intergenerational mobility and output at the same time. However, Piketty's conclusions are ambiguous when persistence is explained by reference group theory. We show that when relative income and leisure are complements reference groups always promote higher effort levels and lead to both higher mobility and output. Ray (2006) argues that it is perfectly possible for an unequal society to create local attainable incentives among the poorest individuals. Affirmative action and public education may be policy tools that could be used to create higher local connectedness and to affect aspiration conformity. We show under what conditions these policies may also contribute to improving mobility when relative groups matter.

The rest of this paper is organized as follows. The next section provides a brief outline on how relative concern has been modelled in economics. The third section focuses on the role of income comparisons and their implications in terms of effort decision and income mobility when we assume forward-looking agents. The fourth section considers backward-looking agents under imperfect information and introduces an updating beliefs rule to describe the long-term effort equilibrium. Finally we conclude.

2 Relative concern in economics

In this paper we define status as relative concern. Postlewaite (1998) and Frank (2005) suggest that evolutionary theory provides a strong argument for an innate concern for relative standing. Agent's relative concern is explained by competition for relative position in their evolutionary past. Hopkins (2008) distinguishes three different evolutionary explanations. The "rivalry story" (the success of others agents reduces own opportunity), the "information story" (the experiences and success of other agents is useful information about potentially profitable activities) and the "perception story" (because preferences are incomplete, relative comparison is a fundamental psychological mechanism to evaluate goods).[3]

Sociologists have a long standing interest in the concept of social status to study social interactions (Weber, 1922). However, this concept has received somewhat less attention in economics.[4] One central issue is whether status is a direct argument of the utility function or its relevance is only instrumental. In this paper we assume that status has intrinsic value and we focus on relative income with respect a reference group.[5] According to the second interpretation, status is relevant because it indirectly affects their opportunities and could be interpreted as an investment decision. In this case, status could be analyzed within the traditional economic paradigm, which assumes agents optimizing with stable preferences (Postlewaite, 1998).

Reference groups are endogenous, and they are likely to depend on several factors. Falk and Knell (2004) argue that individuals choose the reference group to balance self-improvement and self-enhancement motives, while Clark and Senik (2010) suggest that reference groups depend on the type of regular social interactions of individuals. The empirical literature regarding the selection process of the reference group is inconclusive. We will abstract from these aspects and assume that the composition of the reference group is exogenous.

We model relative concern by making assumptions about the effect of the income gap y^R between own y and reference group income y^{RG}, $y^R = y - y^{RG}$, on own utility.[6] Previous studies typically follow the standard assumptions of neoclassical theory that suggest utility to increase with y^R, i.e. $\frac{\partial U(\cdot)}{\partial y^R} > 0$,[7] and marginal utility to diminish with relative income ($\frac{\partial^2 U(\cdot)}{\partial^2 y^R} < 0$) when $y^{RG} < y$. However, there is less agreement on the sign of the second derivative with respect to relative income for agents with relative deprivation ($y^{RG} > y$). Vendrik and Woltjer (2007) argue that the objective function could be convex or concave in relative income, for those agents. The standard assumption of diminishing marginal utility of income in neoclassical theory implies a concave objective function in relative income, while prospect theory (Kahneman and Tversky, 1979), leads to a concave

[3] An alternative explanation would be that relative concern arises from current social arrangements. The nature of economic competition of institutions induces individuals to make relative comparisons (Hopkins, 2008).

[4] See Heffetz and Frank (2011) for a review of the literature.

[5] Rank of the individual in the outcome distribution of the reference group is also used to model relative concern or status (Layard, 1980; Robson, 1992; Clark et al., 2009a; 2009b).

[6] See Hopkins (2008) for a review of models of relative income concern. Since we study reference groups, we model relative concern relative to a reference point. Alternative modelling strategies use inequality aversion to model relative concern. For instance, Alesina and Giuliano (2011) use non-self-centered inequality aversion, which results from the externalities that inequality generates or from agent's views about fairness. Fehr and Schmidt (1999), however, assume self-centered inequality aversion where agents dislike others having more (envy) but where low income for others reduces own utility (compassion).

[7] Alternative options have been proposed, mainly for incomes above the reference group income, i.e. $y^R > 0$. Duesenberry (1949) argues that poorer individuals are negatively influenced by the income of their richer peers, while the opposite is not true. This implies $\partial U(.)/\partial yR = 0$ if $y^R > 0$, while $\partial U(.)/\partial y^R > 0$ if $y^R < 0$. It has also been argued that for compassionate individuals own utility increases when there is an improvement in the income of those agents below them, which implies $\partial U(.)/\partial y^R < 0$ if $y^R < 0$.

objective function in relative income, reflecting diminishing marginal sensitivity to larger deviations from the reference group income ($\frac{\partial^2 U(\cdot)}{\partial^2 y^R} > 0$) –see Figure 1. In modeling the agent's objective function below, we will show that mobility implications of reference groups depend crucially on whether standard or prospect theory assumptions are used.

3 A model of effort choice when reference groups matter

3.1 Basic assumptions

We assume an economy made up of a continuum of agents $I = [0, 1]$, who can be divided into two social backgrounds, lower class origin (I_L; i.e. whose parents' income level was y_0) and upper class origin (I_U, i.e. whose parents' income level was y_1). In this economy the agent's income is a random variable and there are two possible income levels, y_0 and y_1 ($0 < y_0 < y_1$ and $\Delta y = y_1 - y_0$). The probability that agent i obtains a high income level depends positively on her ability ($B \geq \beta > 0$), her effort ($e_i \in (0, \bar{E})$) and luck (π, with $\pi \in (0, 1]$).[8] Furthermore this probability is conditioned by social origin and it is given by,

$$\begin{aligned} Pr(y_i = y_1 | I_L) &= \pi + \theta \beta e_i \\ Pr(y_i = y_1 | I_U) &= \pi + \Delta\pi + \theta \beta e_i \end{aligned} \tag{1}$$

where $Pr(.)$ defines the probability of the event in brackets occurring, $\Delta\pi$ captures previous inequality between agents from different social backgrounds, and $\theta > 0$ is the same for all agents and measures the extent to which higher effort and higher ability translate into higher probability of high income.[9] Because they receive inheritance ($\Delta\pi{>}0$) from previous generations, for the same effort (and ability) the expected probability of economic success is higher for agents from origin I_U than for those from origins I_L.

Furthermore, we assume $\pi + \Delta\pi + \theta B \bar{E} < 1$.[10]

3.2 Agents' objective function

To discuss how optimal effort decisions are affected by income comparisons, we include an additional argument in the standard individual utility function: the self-perceived

[8] There is an exogenous maximum effort level $\bar{E} > (1 - \alpha) c \theta \beta_M \Delta y$. This allows us avoid corner solutions in probabilities. Furthermore, without status motives the unique equilibrium effort level will be lower than $\bar{E}$.

[9] $\Delta\pi$ explains the inequality of family transmitted human capital and/or inequality of collateral in case of credit constrains (Piketty, 1998).

[10] This assumption guarantees that the probability of economic success falls strictly between 0 and 1.

valuation of their own relative position. Therefore, the objective function of agent i is given by,

$$U_i(y_i, y_i^R, e_i) = (1-\alpha)y_i + \alpha G(y_i^R) - C(e_i) \tag{2}$$

where U_i is the utility function for agent i and $\alpha \in [0,1]$. Agents enjoy own income (y_i) for consumption reasons, and dislike effort e_i because they enjoy leisure (agents perceive that effort is a cost defined by the function $C(e_i) = e_i^2/2c$, where $(1/c)$ is the marginal cost of effort and $c > 0$).[11] For simplicity, we first assume that the utility function is additively separable, and that the status motive is a direct argument of the utility function due to its intrinsic value, where $0 < \alpha < 1$ measures the extent to which agents care about it. Agents care about their relative deprivation (RD) which arises from a comparison between their income and that of their reference group, and they dislike unfavorable income comparisons.[12] The function $G(y_i^R) = G(y - y_i^{RG})$ is an attempt to formalize the discussion of how reference group income and relative concern affect an agent's utility, where y_i^R represents the difference between own income (y_i) and expected reference group income (y_i^{RG}).[13]

Following our discussion in section 2, $G(y_i^R)$ is a continuous function defined as,

$$G(y_i^R) = \begin{cases} G(y_i^R) = G(y_i^R) < 0;\ G_{y_i^R}(\cdot) > 0; & if\, y_i^R < 0 \\ G(y_i^R) = 0 & if\, y_i^R \geq 0 \end{cases} \tag{3}$$

where $G_{y_i^R}(.)$ is the first derivative of $G(\cdot)$ with respect to y_i^R.

As in previous studies, we assume asymmetry in income comparisons.[14] This assumption recognizes that agents are upward looking when making comparisons and that the envy effect dominates relative comparisons. Agents care about having a small gap between their income and their reference group income, but relative concern disappears when this

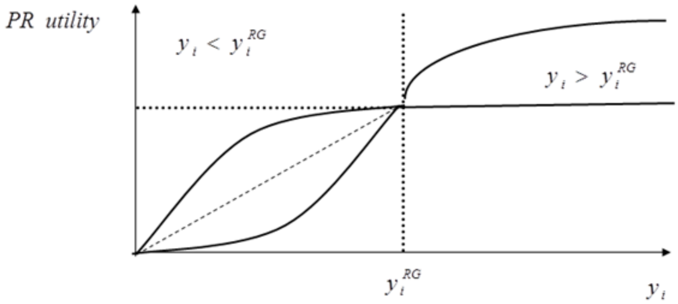
[11] Since we focus on the incidence of relative income on effort decisions, we assume a linear relationship between absolute income and utility, for simplicity. Other studies assume non-linear relationships, and explain the implications for income mobility (Lewis and Ulph, 1998; Antman and McKenzie; 2007; Carter and Barrett; 2006).

[12] Since our concern is the individual's income gap, we do not consider non-self-centered inequality aversion in the objective function.

[13] We assume a cardinal perspective of relative income concern, a decision based on previous papers. This assumption is also related with aspiration models. As noted in Bilancini and Boncinelli (2008), cardinal and ordinal approaches have different implications. Nevertheless, it must be emphasized that assumptions about second and third derivatives of $G(\cdot)$ incorporate ordinal concern (Kolm, 1976a; 1976b).

[14] Other studies have already used this assumption. Stark et al. (2012) used the same assumption to formalize the link between human capital choices and social location choices. Bowles and Park (2005) used it to model the "Veblen effect". Genicot and Ray (2017) also suggest upward looking aspirations formation to describe the relationship between social interaction and aspiration formation. Dalton et al. (2016) use a similar framework to explain aspiration failure. Dusenberry (1949) postulated and tested the hypothesis that relative income comparisons are asymmetric. Finally, this assumption is empirically supported by Bowles and Park (2005), Stuzter (2004) and Ferrer-i-Carbonell (2005).

Figure 1: Relative concern functional form



gap is positive.[15]

3.3 The reference group income

We consider exogenously defined reference groups. The composition of reference groups defines a reference income level and agents care about the gap between their own income and their reference income. The fraction P_i and $(1 - P_i)$ of agents from origin I_U and I_L respectively form the reference group of agent i. Each agent i knows her P_i, which is a random variable with distribution function $F(P_i)$ for all $P_i : 0 \leq P_i \leq 1$.[16] Agent i from social origin I_L compares only with her peers when $P_i = 0$, and she only compares with upper-class agents when $P_i = 1$. As a result, the expected income of the reference group, y^{RG}, is defined as $y^{RG} = P_i(E(y|I_U)) + (1 - P_i)E(y|I_L)$.

As stated above, we further assume that social comparisons are upwards. In our setting this means that richer agents compare only with their peers. That is, we assume that $P_i = 1$ for agents from upper-class origins, which is consistent with previous findings that suggest income comparisons not to be downward-looking (e.g. Ferrer-i-Carbonell, 2005).[17]

3.4 Informational assumptions

The following informational assumptions are based on previous papers (Piketty, 1995; 1998; Breen and García-Peñalosa, 2002) and some stylized facts concerning social inter-

[15] Because we focus on the decision of agents with relative deprivation, to simplify, in this section we assume that pride and compassion effects are 0 or offset each other. Even though this assumption simplifies the analysis, it is worth noting that this only affects the decision of agents without relative deprivation. If pride effect dominates, the relative income effect motivates a high effort of agents without relative deprivation.

[16] Reference groups may be the result of individual choice (Falk and Knell, 2004) or may be conditioned by individual circumstances. We adopt the latter and assume that reference groups are related with identity factors and social norms. This idea is consistent with the assumptions about the aspiration window suggested by Ray (2006).

[17] This assumption is not essential. The conclusions of section 3.5, which assumes forward-looking agents, do not change if we assume that $0 \leq P \leq 1$ for agents from upper-class origins.

actions, individual beliefs, and income inequality.

- *IA.i.* Agents have perfect information about the parameters that determine the probability of economic success (π, $\Delta\pi$ and θ).
- *IA.ii.* Ex-ante, agents do not have any information about their ability β_i (nor do they know their relative ability) and they assume the mean β_M of the ability distribution $f(\beta_i)$, with $0 < \beta_i \leq B$.
- *IA.iii.* Other individuals' effort levels, their ability, and their P_i are not publicly observable; everybody expects agents from lower-class origin to exert effort $0 \leq e_L^b \leq E$ and those from upper-class origin to exert effort $0 \leq e_U^b \leq E$.
- *IA.iv.* Finally, we assume that the expected income of agents from upper-class origin is at least equal to the expected income of agents from lower-class origin: since inheritance is positive only for upper-class agents ($\Delta\pi > 0$), $(\pi + \theta\beta E)y_1 + (1 - \pi - \theta\beta E)y_0 = Max(E(y_i \mid I_L) \leq E(y_i \mid I_U)$.

Piketty (1998) and Breen and Garcia-Peñalosa used assumption *IA.i*, while Piketty (1995, 1998) assumed *IA.ii*. Previous papers assume $P_i = 0$, while unobservability of effort (*IA.iii*) is a standard assumption.

As a result, expected income for lower-class and upper-class origin agents is respectively defined as follows,

$$\begin{aligned} E(y_i|I_L) &= (\pi + \theta\beta_M e_L^b)y_1 + (1 - \pi - \theta\beta_M e_L^b)y_0 \\ E(y_i|I_U) &= (\pi + \Delta\pi + \theta\beta_M e_U^b)y_1 + (1 - \pi - \Delta\pi - \theta\beta_M e_U^b)y_0 \end{aligned} \tag{4}$$

The expected relative deprivation depends on the expected income of agents with different social origins and on the composition of reference groups. Consider first the case of agent i from lower-class origin (I_L). The ex-ante expected relative deprivation is defined as,

$$E(y_i^R \mid I_L) = E(y_i \mid I_L) - E(y_i^{RG}) =$$

$$\underbrace{P_i}_{Composition} \underbrace{[(E(y \mid I_L) - E(y \mid I_U)]}_{\substack{Expected\, income\, gap \\ between\, agents\, I_L and\, I_U}} + \underbrace{E(y_i \mid I_L) - E(y \mid I_L)}_{\substack{Expected\, income\, gap \\ of\, agent\, i,\; with\, his\, peers}} = \xi(e_i, e_L^b, e_U^b, P_i) \tag{5}$$

where $E(y_i \mid I_L)$ is the expected income of agent i, given that she is I_L, and $E(y \mid I_L)$ is the expected income for agents from origin I_L, which was defined in equation 4. Relative deprivation is composed of three terms: the composition of the reference group (P_i), the expected gap between agents from lower- and upper-social origin ($E(y \mid I_L) - E(y \mid I_U)$), and the expected gap with respect to peer's income. Observe that relative deprivation has a "random component", P_i, and an inheritable component, the expected income conditional on the origin. P_i also could be interpreted as the quality of information about the peer group's income.

For agents from origin I_U, the expected relative deprivation is defined as,

$$E(y_i^R \mid I_U) = E(y_i \mid I_U) - E(y \mid I_U) \qquad (5.b)$$

Finally, assumption ($IA.iv$) implies that differences in expected effort between lower- and upper-class origin individuals never outweigh the effect of previous inequality ($\Delta\pi$). Regardless of the value of P_i expected relative income is smaller for agents from origin I_L than for agents from origin I_U ($E(y_i^R \mid I_L) \leq E(y_i^R \mid I_U)$). **Martin, is this last sentence correct? The previous version said the opposite**

3.5 Agents' effort decisions

We assume that agents live one period, are rational, and maximize their expected utility conditioned on the parameters of the Economy and their beliefs. For agent i from lower-class origin, the optimization problem is defined as,

$$\begin{cases} Max_e\, E\left[U_i(y_i, y_i^R, e_i) \mid I_L)\right] = (1-\alpha)E\left[y_i \mid I_L\right] + \alpha E\left[(G(y_i^R \mid I_L))\right] - C(e_i) \\ \qquad s.t.\;\; E(y_i^R \mid I_L) = \xi(e_i, e_L^b, e_U^b, P_i) \end{cases} \qquad (6)$$

As a benchmark, we consider that e_L^b and e_U^b are exogenous and agents know their values (each agent takes others' effort as given), public beliefs are always shared (*Assumption IA.iii*). In this case, the utility-maximizing effort level for a lower-class origin agent $e_{Leq}(P_i)$ is given by

$$\begin{cases} e_{Leq}(P_i, e_L^b, e_U^b) = ArgMax_{e_i>0} = (1-\alpha)(y_0 + (\pi + \theta\beta_M e_i)\Delta y) + \alpha E\left[(G(y_i^R \mid I_L))\right] - e_i^2/2c \\ \qquad s.t.\;\; y_i^{RG} = y_0 + P_i(\pi + \Delta\pi + \theta\beta_M e_U^b)(\Delta y) + (1-P_i)(\pi + \theta\beta_M e_L^b)(\Delta y) \end{cases} \qquad (7)$$

The first order condition is

$$e_{Leq}(P_i, e_L^b, e_U^b) = \begin{cases} e_{Leq}^{*} = (1-\alpha)c\theta\beta_M \Delta y & if\, E(y^R \mid I_L) \geq 0 \\ e_{Leq}^{**} = e_{Leq}^{*} + \alpha c\theta\beta_M \Delta y G_{y^R}(y_{Leq}^R \mid I_L) & if\, E(y^R \mid I_L) < 0\, \&\, e_{Leq}^{**} < \bar{E} \\ e_{Leq} = \bar{E} & if\, E(y^R \mid I_L) < 0\, \&\, e_{Leq}^{**} \geq \bar{E} \end{cases} \quad (8)$$

For agents from origin I_U the utility-maximizing effort level e_{Ueq} is given by

$$\begin{cases} e_{Ueq}(e_U^b) = ArgMax_{e_i > 0} = (1-\alpha)(y_0 + (\pi + \Delta\pi + \theta\beta_M e_i)\Delta y) + \\ \alpha E\left[(G(y_i^R \mid I_U))\right] - e_i^2/2c \\ S.a. y_i^{RG} = y_0 + (\pi + \Delta\pi + \theta\beta_M e_U^b)(\Delta y) \end{cases} \quad (7.b)$$

and the first order condition is

$$e_{Ueq}(e_U^b) = \begin{cases} e_{Ueq}^{*} = (1-\alpha)c\theta\beta_M \Delta y & if\, e_U^b < e_{Ueq}^{*} \\ e_{Ueq}^{**} = e_{Ueq}^{*} + \alpha c\theta\beta_M \Delta y G_{y^R}(y_{Ueq}^R \mid I_U) & if\, \mathrm{E} > e_U^b > e_{Ueq}^{*} \\ e_{Ueq} = \bar{E} & if\, e_U^b = \bar{E} \end{cases} \quad (8.b)$$

The second order condition $\left[\alpha(\theta\beta_M \Delta y)^2 G_{y^R y^R}(y_{eq}^R \mid I_L) - \frac{1}{c} < 0\right]$ holds because of the concavity of $G(y^R)$ (in accordance with standard assumptions) and of $c(e)$. Hence $e_{Leq}(P_i)$ and e_{Ueq} constitute optimum solutions.

All agents with the same reference group will choose the same optimal effort. Namely, agents from origin I_L (I_U) and the same P_i, will choose the same optimal effort $e_{Leq}(P_i) = e_{eq}(P_i)$, where index i identifies the reference group composition. Effort depends on reference group composition for agents with relative deprivation, but not for agents with relative affluence. Note then that agents from the same origin may choose different effort levels because the composition of their reference groups differ. This result deviates from Piketty (1998), where all agents from the same origin arrive to the same long-term effort level. Finally, agents with relative affluence will always exert less effort than those with relative deprivation ($e_{Leq}^{**} > e_{Leq}^{*}$ because $\alpha a\theta\beta_M \Delta y G_{y^R}(y_{Leq}^R \mid I_L) > 0$ and $e_{Ueq}^{**} > e_{Ueq}^{*}$ because $\alpha a\theta\beta_M \Delta y G_{y^R}(y_{Ueq}^R \mid I_U) > 0$).

The effect of reference groups on effort

How do optimal effort levels react to changes in reference group incomes? The partial derivative of optimal effort levels, derived in equation (8), with respect to y^{RG} provides the answer. We focus on agents from low social origin and expected relative deprivation ($E(y_i^R \mid I_L) < 0$), but the analysis is analogous for agents I_U.

$$\partial e^{**}_{Leq}/\partial y^{RG} = \frac{-\alpha c\theta\beta_M \Delta y G_{y^R y^R}(.)}{1-\alpha c(\theta\beta_M\Delta y)^2 G_{y^R y^R}(.)}\, if\, E(y_i^R \mid I_L) < 0\, and\, e^{**}_{Leq} < E \qquad (9)$$

Since we start assuming that $G(\cdot)$ is concave (i.e. $G_{y^R y^R}(\cdot) < 0$), this expression is always positive suggesting the complementarity between effort and reference group income. The derivative is zero when effort reaches its maximum level ($e^{**}_{Leq} = E$). As a result, richer (or more demanding) reference groups provide higher effort incentives for lower-class agents. This effect is larger when agents care a lot about their relative position (high α) and when they are more sensitive to changes in relative deprivation (i.e. high $G_{y^R y^R}(\cdot)$).

When $P_i \neq 0$ agents from lower-class origins have high economic incentives to increase the amount of effort. This effect is stronger the higher P_i, e^b_U and e^b_L. Importantly, higher expected inequality (higher Δy and $\Delta\pi$) also creates incentives to work hard and exert more effort for people with expected relative deprivation (Alessina and Guliano, 2011). These incentives, however, disappear when $E(y_i \mid I_L) \geq E(y_i^{RG})$, as $e^*_{Leq} = (1-\alpha)c\theta\beta_M\Delta y$.

Note that the concavity assumption of $G(\cdot)$ models the encouragement effect, but does not capture frustration or complacency effects. Several arguments and evidence, however, challenge the encouragement effect. For instance, it has been argued that increased relative deprivation may increase the cost to access the resources needed to participate in social activities (Vendrik and Vendrik, 2007; Sen 1985b), which may affect the marginal cost of effort and effort decisions. This idea is consistent with the social psychology literature on the relevance of social emotions to explain individual behavior. For example, Kuziemko et al., (2014) suggest that individuals are likely to face little shame when near the mean, but shame may increase quickly when they move towards the bottom of the distribution. People may also have different views about what originates illegitimate or unfair inequality, and this is likely to influence the effect of inequality on effort (Besley, 2017). In our model, higher $\Delta\pi$ represents the larger importance of inheritance in the income generating process, which could be perceived as an unfair circumstance. Experimental evidence has shown that agents are willing to punish unfair situations, even at some immediate cost to themselves (Dawes et al., 2007; Henrich et al., 2010; Fehr and Hoff, 2011). As a result, it is debatable that higher ex-ante inequality $\Delta\pi$ always motivates higher effort. Based on these arguments, it is possible to argue that people could change their perception of the cost of effort and their motivations, because they face the increasing cost of relative deprivation or because they think that the initial distribution is unfair.

To accommodate these arguments in our framework, we assume that $G(\cdot)$ is convex (i.e. $G_{y^R y^R}(\cdot) > 0$) when individuals face relative deprivation ($E(y^R) < 0$), as suggested by prospect theory, which reflects diminishing marginal sensitivity to larger deviations from the reference group income (see section 2).[18] This assumption is also supported by

[18]Note that assuming $G_{y^R y^R}(\cdot) > 0$ when $E(y^R) < 0$, $G(.)$ satisfies three key assumptions of prospect

Kuziemko et al., (2014), who argue that in the presence of last-place aversion, the utility of agents in the bottom of the income distribution may be convex with respect to relative position. Now, when $G_{y^R y^R}(.) > 0$ and $\alpha\theta\beta_M \Delta y \left[G_{y^R y^R}(.) < \frac{1}{c}\right]$, $\partial e^{**}_{Leq}/\partial y^{RG}$ is negative, and the optimality condition still holds in a range of values of y^R.[19] In other words, more demanding reference groups lead to lower effort. We have arrived at the following proposition:

Proposition 1. When $E(y^R) < 0$, under additive comparisons and asymmetry in income comparisons:

(i) The reference income effect increases the optimal level of effort chosen by an agent with relative deprivation compared to an agent with relative affluence ($e^*_{Leq} < e^{**}_{Leq}$).

(ii) When the utility function is convex in relative income ($G_{y^R y^R}(.) < 0$), higher reference income always leads to additional effort ($\partial e^{**}_{Leq}/\partial y^{RG} > 0$ with $e^{**}_{Leq} < \bar{E}$).

(iii) When the utility function is concave in relative income ($G_{y^R y^R}(.) > 0$), higher reference income always leads to lower effort $\partial e^{**}_{Leq}/\partial y^{RG} < 0$ with $\bar{E} > e^{**}_{Leq} > e^*_{Leq}$).

Proof. direct from eq. (9) and the functional form of $G(\cdot)$.

In sum, assumptions about the sign of $G_{y^R y^R}(\cdot)$ reflect the difference between prospect and standard theory, and are central in explaining the effect of reference groups, while allowing us to model both the encouragement effect and the frustration or complacency effects.

The role of the reference group on income mobility

Next we discuss whether relative concern generates differences in effort decisions between agents from different social origins. To simplify the discussion we assume $G_{y^R y^R}(.) < 0$, which represents the most optimistic case, since inequality encourages higher effort.

When $\alpha = 0$ (i.e. with no relative concerns), equations (8) and (8.b) trivially define a unique equilibrium where all agents exert the same effort. However, when $\alpha \neq 0$, equilibrium effort depends on P_i, e^b_U and e^b_L.

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theory (a) reference dependence; (*b*) asymmetric valuation between gains and losses; and (*c*) principle of diminishing sensitivity.

[19] The expression $\alpha\theta\beta_M \Delta y \left[G_{y^R y^R}(.) < \frac{1}{c}\right]$ implies that effort is always perceived as a cost. In other words, an increase in the marginal utility due to a decrease in relative deprivation is lower than the increase in the marginal cost due to higher effort.

There is one extreme case, when $e^b_U = e^b_L = \bar{E}$, then $e_{Leq} = e_{Ueq} = \bar{E}$. This result is consistent with "self-fulfilling beliefs". For agents from either origin, I_L or I_U, expected efforts are high and agents choose high effort. Observe that, although $e_{Leq} = e_{Ueq}$, both scenarios establish that $E(y_i \mid I_L) < E(y_i \mid I_U)$ and $E(y^R_i \mid I_L) \leq E(y^R_i \mid I_U)$.

Differences in expected efforts and relative deprivation yields different optimal choices. Let us first consider $e^b_U < e^b_L$. If $P_i \neq 0$, the effort of agents from origin I_L is higher than the effort of agents from origin I_U ($e_{Leq}(P_i) > e_{Ueq}$). In this case, increased relative deprivation increases the optimal level of effort chosen by an agent from low social origin, for all positive values of P_i (since $G_{y^R}(y^R) > 0$, then $e^*_{Ueq} = e^*_{Leq} < e^{**}_{Leq}$). However, if agents from origin I_L only compare with their peers, $P_i = 0$, effort is the same across agents from different social origins.

Alternatively, if $e^b_U > e^b_L$, differences in effort across agents from different social origins depend essentially on P_i. There is a P^* such that $\frac{-P^*\Delta\pi}{1-P^*} = \theta\beta_M \left[e^b_L - e^b_U\right]$, which leads to effort being the same across social origins ($e_{Leq}(P^*) = e_{Ueq}$), as $E(y^R \mid I_L) = \xi(e_i, e^b_L, e^b_U, P^*) = E(y^R \mid I_U)$. However, if $P_i > P^*$, $e_{Leq}(P_i) \geq e_{Ueq}$. That is, a more demanding reference group (higher P_i) leads to higher effort for agents from origin I_L. On the contrary, if P_i is lower than P^*, agents from origin I_L compare mainly with their peers, and exert lower effort than agents from from origin I_U, $e_{Leq}(P_i) < e_{Ueq}$.[20]

3.6 When relative effort matters

In the previous section, reference groups enter our discussion only through relative income deprivation. However, reference group theory considers relative deprivation as a social and psychological experience, in which individuals take the standards of other individuals as a comparative "frame of reference". This defines not only "the patterns of expectations", but also the perception of "comparable sacrifice" and it thus contributes to explaining why attitudes differ among individuals (Merton, 1953; Clark and D'Ambrosio, 2014; Heffetz and Frank, 2011). To address this issue we leave aside the additive comparisons assumption and include a more general function $G(y^R_i, e_i)$, with $G_{y^R_i, e_i}(\cdot) \neq 0$, which includes both relative income and relative effort (with respect to relative deprivation). This function incorporates the part of the cost of effort that is cultural and endogenous, while $C(e_i)$ is the part of effort that is exogenous to the relative situation.[21] As a result, this function

[20] Observe that when $P_i = P^*$, $G(y^R \mid I_L) = G(y^R \mid I_U)$, and then $e_{Leq}(P^*, e^b_L, e^b_U) = e_{Ueq}$. But when $P_i > P^*$(or $P_i < P^*$), $E(y^R \mid I_L) < E(y^R \mid I_U)$ (or $E(y^R \mid I_L) > E(y^R \mid I_U)$ and $G(y^R \mid I_L) > G(y^R \mid I_U)$ (or $G(y^R \mid I_L) < G(y^R \mid I_U)$.

[21] Based on the notions of cognitive dissonance, relative deprivation and social comparison, Festinger (1957) argues that individuals compare their own input-to-output ratio with respect to a reference level. According to equity theory, if the comparison is perceived as "unfair", the individual may be motivated to change his behavior and restore his cognitive perception of equality (Adams, 1965). Kandel and Lazear (1992) or Akerlof and Kranton (2005), incorporate the notion of social norms and analyze how it affects work incentives.

considers the way in which relative deprivation affects the perception of effort and how effort affects the sensitivity to relative deprivation.[22] This way we capture the idea that reference groups establish the "effort norm", which could affect individual motivation. We include the function $G(y_i^R, e_i)$ in the agent's objective function and arrive at,

$$U_i(y_i, y_i^R, e_i) = (1-\alpha)y_i + G(y_i^R, e_i) - C(e_i) \tag{10}$$

Following the standard assumption, we assume that $G(y_i^R, e_i)$ is decreasing and concave in its first argument. However, in the second argument the situation is more flexible, and its functional form allows us to model different individual responses and include some convex parts of function $G(\cdot)$.

$$G(y_i^R) = \begin{cases} G(y_i^R, e_i) = G(.) > 0;\ G_{y_i^R}(.) > 0;\ G_{y_i^R y_i^R}(.) < 0 & if\ y_i^R < 0 \\ G(y_i^R) = 0 & if\ y_i^R \geq 0 \end{cases} \tag{11}$$

If we assume forward-looking agents and consider eq. (10) in the optimization problem defined in equations (6) and $(6.b)$, we can derive new optimal effort conditions of agents from origin I_L and I_U.

$$e_{Leq}(P_i, e_L^b, e_U^b) = \begin{cases} e_{Leq}^* = (1-\alpha)c\theta\beta_M\Delta y & if\ E(y^R \mid I_L) \geq 0 \\ e_{Leq}^{**} = e_{Leq}^* + \alpha a\theta\beta_M\Delta y G_{y^R}(.) + a\alpha\left[G_e(.)\right] & if\ E(y^R \mid I_L) < 0\ \&\ e_{Leq}^{**} < \bar{E} \\ e_{Leq} = E & if\ e_{Leq}^{**} \geq \bar{E} \end{cases} \tag{12}$$

$$e_{Ueq}(e_U^b) = \begin{cases} e_{Ueq}^* = (1-\alpha)c\theta\beta_M\Delta y & if\ e_U^b < e_{Ueq}^* \\ e_{Ueq}^{**} = e_{Ueq}^* + \alpha a\theta\beta_M\Delta y G_{y^R}(.) + a\alpha\left[G_e(.)\right] & if\ \mathrm{E} > e_U^b > e_{Ueq}^* \\ e_{Ueq} = \bar{E} & if\ e_U^b = \bar{E} \end{cases} \tag{12.b}$$

We assume that the problem has an optimal solution and the following second order

[22] To make this assumption a little more concrete, consider an example of the function $G(y_i^R, e_i)$, $G(y_i^R, e_i) = g(y_i^R)v(e_i)$, with $g(y_i^R) > 0$, $g'(y_i^R) > 0$, $g''(y_i^R) < 0$ and $v(e_i) > 0$. Note that $v(e)$ is constant and equals 1 in the basic model (section 3.5). By making explicit assumptions about $v(e_i)$, we clarify the exact nature of the tastes required to explain a particular behavior. On the one hand, when effort increases, the marginal utility of relative deprivation in the reference group will decrease. Namely $v'(e_i) > 0$, which implies $G_e(.) > 0$. On the other hand, the sensitivity for relative deprivation might decrease with higher effort, if $v'(e_i) < 0$, which implies $G_e(.) < 0$. This function also captures how relative deprivation affects the perception of the cost of effort. For example, perception of the cost of effort could be lower when relative deprivation is low, because agents believe that reference group income is an achievable outcome and they are motivated ($v'(e_i) < 0$). Alternatively, given a high relative deprivation, when effort is very high, agents could perceive that the goal is unattainable, they are discouraged and perceive that effort is less effective (or more costly, $v'(e_i) > 0$).

condition always holds,

$$\alpha G_{ee}(.) + 2\alpha\theta\beta_M \Delta y G_{y^R e}(y_i^R, e_i) < \tfrac{1}{c} - \alpha\theta^2\beta_M^2 \Delta y^2 G_{y^R y^R}(y_i^R, e_i) \tag{13}$$

As a result, $e_{Leq}(P_i, e_L^b, e_U^b)$ and $e_{Ueq}(e_U^b)$ constitute optimum solutions. The FOC remains unchanged for agents with lower reference group income than own income, i.e. when $E(y^R \mid I_L) \geq 0$ "relative deprivation" has no effect on optimal effort level. However, this condition changes when $E(y^R \mid I_L) < 0$. If we only focus on interior solutions, an agent from origin I_L and I_U will choose the level of effort $e_{Leq}^{**}(P_i, e_L^{b**}, e_U^{b**})$ and $e_{Ueq}^{**}(e_U^{b**})$ respectively.[23] In this case, the sign of $G_e(.)$ characterizes the agent's response to relative deprivation and to reference group income. We first examine the impact of relative deprivation on effort. To this end, in Proposition 2 we compare optimal effort of individuals in relative deprivation with individuals in relative affluence, who are otherwise identical.

Proposition 2. When $E(y^R) < 0$, under non-additive comparisons and asymmetry in income comparisons, we have:

> *Positional self-encouraged agent*, when $G_e(\cdot) \geq 0$ (Condition I), relative deprivation increases the optimal level of effort chosen by an agent with relative deprivation compared to the effort chosen by an identical agent with no relative deprivation (with $e_{Leq}^{**} \leq \bar{E}$).
>
> *Positional stimulated agent*, when $G_e(\cdot) < 0$ and $-G_e(\cdot) > \theta\beta_M \Delta y G_{y^R}(\cdot)$ (Condition II), relative deprivation increases the optimal level of effort chosen by an agent with relative deprivation compared to the effort chosen by an identical agent with no relative deprivation (with $e_{Leq}^{**} \leq \bar{E}$).
>
> *Positional discouraged agent*, when $G_e(\cdot) < 0$ and $G_e(\cdot) < \theta\beta_M \Delta y G_{y^R}(\cdot)$ (condition III), relative deprivation decreases the optimal level of effort chosen by an agent compared to the effort chosen by an identical agent with no relative deprivation (with $e_{Leq}^{**} \leq \bar{E}$).

Proof. direct from equation (12) and the functional form of $G(y_i^R, e_i)$.

When $G_e(\cdot) \geq 0$, the equilibrium effort level e_{Leq}^{*} is always lower than e_{Leq}^{**}. Under condition I, given the same level of effort, agents with relative deprivation perceive a lower cost for additional relative effort and, therefore, they exert higher effort. Here, function $G_e(\cdot)$ can be interpreted as implying that agents get utility from relative effort.

[23] Note that non-interior solutions are analogous to those discussed in the previous section. e_L^{b**} and e_U^{b**} denote equilibrium effort beliefs.

Not surprisingly, then, this spurs more effort (self-motivated effect).[24]

When $G_e(\cdot) < 0$, relative effort is always a cost. Given an expected income gap, relative deprivation generates lower utility among those agents who have made greater effort. However, if $-G_e(\cdot) < \theta\beta_M \Delta y G_{y^R}(\cdot)$ the larger disutility of high relative effort is compensated by a lower relative income gap. Therefore, the encouragement effect dominates because there is high opportunity for income mobility ("relative effort pays" because $\theta\beta_M\Delta y$ is large enough).

However, if $-G_e(\cdot) > \theta\beta_M \Delta y G_{y^R}(\cdot)$, the disutility of effort is larger than the gain in utility that results from the reduction in the relative income gap. Effort does not pay, and individuals in relative deprivation are discouraged, so they reduce relative effort. Now $e^*_{Leq} > e^{**}_{Leq}$.

Positional self-encouraged and stimulated agents increase upward mobility because their income increases as a result of higher relative effort. In contrast, positional discouraged agents reduce upward mobility.

We first examine the impact of relative deprivation on effort. To this end, in Proposition 2 we compare optimal effort of individuals in relative deprivation with individuals in relative affluence, who are otherwise identical.

We next explore what is the effort response to an exogenous change in y^{RG} among agents with relative deprivation, when relative effort matters. We are now concerned with the size of the negative income gap rather than with there being a negative income gap –which is what Proposition 2 addresses. By differentiating the individual's first order condition for the choice of effort we find the following expression:

$$\partial e^{**}_{Leq}/\partial y^{RG} = \frac{\alpha c\left[-\theta\beta_M \Delta y G_{y^R y^R}(.) - G_{y^R e}(.)\right]}{1-\alpha c G_{ee}(.) - \alpha c(\theta\beta_M\Delta y)^2 G_{y^R y^R}(.) - 2\alpha c\theta\beta_M\Delta y G_{y^R e}(.)} \tag{14}$$

The expression in the numerator of eq. (14) determines the sign of $\partial e^{**}_{Leq}/\partial y^{RG}$ (the denominator is positive due to the second order condition). Since we assumed $G_{y^R y^R}(.)$ to be negative, the sign of the numerator depends on the sign of $G_{y^R e}(\cdot)$. This sign indicates whether the inverse of effort (leisure) and relative income are complements ($G_{y^R e}(\cdot) \leq 0$) or substitutes ($G_{y^R e}(\cdot) > 0$).[25] If they are complements a larger income gap induces higher effort. If they are not, the sign of $\partial e^{**}_{Leq}/\partial y^{RG}$ is ambiguous, and it depends on the magnitude of $\theta\beta_M\Delta y G_{y^R y^R}(\cdot)$, namely, the sign depends on relative rewards and ex-ante inequality. We express these ideas more precisely in proposition 3:

[24] Kandel and Lazear (1992) use a similar argument to explore how peer pressure operates on worker effort. They suggest that the peer pressure function can be interpreted as implying that workers get utility from effort.

[25] These ideas are used in Bowles and Parker (2005) to discuss the importance of the "Veblen effect" in the individual's allocation of time between labour and leisure.

Proposition 3. When $E(y^R) < 0$, under non-additive comparisons and asymmetry in income comparisons, we have:

Income-gap self-encouraged agent, when $G_{y^R e}(\cdot) \leq 0$ (Condition IV), higher reference income leads to additional effort ($\partial e^{**}_{Leq}/\partial y^{RG} > 0$ with $e^{**}_{Leq} \leq \bar{E}$).

Income-gap stimulated agent, when, $G_{y^R e}(\cdot) > 0$ and $-\theta\beta_M \Delta y G_{y^R y^R}(\cdot) > G_{y^R e}(\cdot)$ (Condition V), higher reference income leads to additional effort ($\partial e^{**}_{Leq}/\partial y^{RG} > 0$ with $e^{**}_{Leq} \leq \bar{E}$).

Income-gap discouraged agent, when, $G_{y^R e}(\cdot) > 0$ and $-\theta\beta_M \Delta y G_{y^R y^R}(\cdot) < G_{y^R e}(\cdot)$ (Condition VI), higher reference income leads to lower effort ($\partial e^{**}_{Leq}/\partial y^{RG} < 0$ with $0 \leq e^{**}_{Leq} \leq \bar{E}$).

Indifferent agents, when $G_{y^R e}(\cdot) > 0$ and $\theta\beta_M \Delta y G_{y^R y^R}(\cdot) = -G_{y^R e}(\cdot)$ (Condition VII), individuals do not respond to changes in reference group income ($\partial e^{**}_{Ueq}/\partial y^{RG} = 0$).

Conditions IV and V establish a positive relation between effort and reference income, but there is a strong difference between them. In the former, higher reference group income decreases the marginal cost of relative effort (relative effort generates utility). As a result, higher reference group income always increases effort levels through two channels, the higher marginal utility of relative income and the lower marginal cost of relative effort. Note that this result is independent of the economic context, namely does not depend on the magnitude of $\Delta\pi$, Δy, θ and β_M.

On the contrary, under conditions V, VI, and VII, relative effort represents a cost, in accordance with standard economic models, but agents' effort responses are ambiguous. Under condition V, the marginal utility of relative income is large enough as to compensate the larger marginal cost of relative effort. Overall, then, an increase in reference income spurs additional effort. Condition VI establishes a negative relation between effort and reference group income ($\partial e^{**}_{Ueq}/\partial y^{RG} < 0$). Now, the marginal utility of relative income is insufficient to compensate the marginal disutility of relative effort, causing a reduction in effort. Finally, under condition VII the marginal utility of relative income just compensates the larger marginal cost of relative effort, so that individuals do not respond to changes in reference group income.

Upward mobility increases when more demanding reference groups result in larger effort, as with income gap self-encouraged and stimulated agents, while it decreases with income-gap discouraged agents.

When relative effort is a cost (i.e. conditions V, VI, and VII) the parameters of economic inequality are relevant in explaining agents' effort responses. Larger returns to effort and ability (θ), expected ability (β), and income premium ($\triangle y$) increases the likelihood of of income-gap stimulated agents and reduces the likelihood of income-gap discouraged agents. This characterizes a positive relationship between inequality and upward mobility, which is consistent with the empirical evidence on the so-called Great Gatsby curve (Krueger, 2012; Corak, 2013).

The effect of effort rewards (θ) and ex-ante inequality ($\triangle\pi$) on effort is more nuanced, as it depends on whether the principle of diminishing transfers holds.[26] If it does, the response of individuals to higher θ is ambiguous. However, this ambiguity disappears when $G_{y^R y^R y^R}(\cdot) \leq 0$. Now, an increase in θ increases expected relative deprivation and the sensibility of marginal utility to relative deprivation. Both effects go in the same direction, so effort increases. Now, however, larger ex-ante inequality ($\triangle\pi$) can reduce or eliminate the incentive to increase effort brought about by a larger θ. Appendix A discusses in more detail the effect of effort rewards and ex-ante inequality on effort and relative deprivation.

Proposition 3 also shows that reference group composition, P, is relevant in explaining effort levels. The relationship between effort and P depends on whether leisure and relative income are complements.[27]

3.7 Long-term equilibrium with perfect forward-looking agents

The previous discussion describes the individual decision process in the most simple case, where agents do not internalize ex-ante beliefs when they take effort decisions. Also, the discussion does not consider the interaction between individual effort decisions and the expected effort of peers (i.e. e^b_U and e^b_L are exogenous). However, own effort decision and their mobility outcomes may affect expected peers' effort. Furthermore, observe that, since the expected income of agents from I_U affects the utility of agents from origin I_L, but the reverse is not true, the former could be interpreted as leaders and the latter as followers (Clark and Oswald, 1998). In line with this ideas, we study the following kind of equilibrium:

Definition A long term equilibrium of the economy is a vector of consistent effort decisions and effort beliefs of agents from low and high social origin, such that $e_{Leq}(P_i)$, e_{Ueq}, e^b_{Ueq} and e^b_{Leq} solve equations (12) and (12.b), for all $i \in I_L$ or $i \in I_U$ and for all t.

To understand the relationship between equilibrium social effort beliefs and individual effort decisions it is convenient to include some additional assumptions. First, we add the following informational assumption.

[26] The Principle of diminishing transfers (i.e. $G_{y^R y^R y^R}(\cdot) > 0$), requires utility to increase more for poorer than for richer individuals in front of the same reduction in the relative income gap of both individuals (Kolm, 1976a; 1976b).

[27] $\partial e^{**}_{Leq}/\partial P = \frac{-\alpha c \Delta y (e^b_L - e^b_U - \Delta\pi)\left[\theta\beta_M \Delta y G_{y_R y_R}(\cdot) + G_{y_R e}(\cdot)\right]}{1+\alpha c G_{ee}(\cdot) + \alpha c(\theta\beta_M\Delta y)^2 G_{y_R y_R}(\cdot) + 2\alpha c\theta\beta_M \Delta y G_{y_R e}(\cdot)}$ and $-\alpha a \Delta y(e^b_L - e^b_U - \Delta\pi) > 0$ (by assumption IA iv).

- *IA.v* We assume agents with fully forward-looking behavior, in the sense anticipating the actions of others when they take effort-decisions (an extreme Cournot-Nash assumption satisfied).

Second, we assume that agents' take decisions in two steps. First, using e_U^b and e_L^b they identify their expected relative deprivation, which reveals their domain in the relative deprivation function, $G(\cdot)$.

$$RD(P_i, e_U^b, e_L^b \mid I_L) \begin{cases} RD = 0\, if\, E(y^R \mid I_L) \geq 0 \\ RD = 1\, if\, E(y^R \mid I_L) < 0 \end{cases} \tag{15}$$

$$RD(e_U^b \mid I_U) \begin{cases} RD = 0\, if\, E(y^R \mid I_U) \geq 0 \\ RD = 1\, if\, E(y^R \mid I_U) < 0 \end{cases} \; (15b) \tag{16}$$

In a second step, they maximize expected utility, taking the reference group income as given and choosing their individual optimal level of effort.

Ex-ante agents share the public beliefs about peers' expected income ($E(y_i \mid I_L) = E(y \mid I_L)$ and $E(y_i \mid I_U) = E(y \mid I_U)$), so that $E(y_i^R \mid I_U) \geq 0$ and $E(y_i^R \mid I_L) \leq 0$. By considering eqs. (15) and (15.b) in the optimization problem defined in eqs. (6) and (6.b), we can derive new optimal effort conditions of agents from origin I_L and I_U.

$$e_{Leq}(P_i) = \begin{cases} e_{Leq}^{*} = (1-\alpha)c\theta\beta_M \Delta y & if\, RD = 0 \\ e_{Leq}^{**} = e_{Leq}^{*} + \alpha c\theta\beta_M \Delta y G_{y^R}(.) + c\alpha\left[G_e(.)\right] & if\, RD = 1\, \&\, e_{Leq}^{**} < \bar{E} \\ e_{Leq} = E & if\, RD = 1\, e_{Leq}^{**} \geq \bar{E} \end{cases} \tag{17}$$

$$e_{Ueq}^{*} = (1-\alpha)c\theta\beta_M \Delta y \, (17b)$$

Because they anticipate the effort of their peers and share social beliefs, agents from origin I_U do not expect to face relative deprivation ($RD(e_U^b \mid I_U) = 0$). Therefore, the equilibrium is $e_{Ueq} = e_{Ueq}^{*} = e_{Ueq}^{b}$ for agents from origin I_U. Note that this can be interpreted as a game with a Nash symmetric equilibrium in which all agents from I_U origin use the same strategy and anticipate expected peer's effort.

However, for agents from origin I_L, the equilibrium is defined by $e_{Leq}(P_i)$ and $e_{Leq}^{b} = \int_0^1 F(P_i) e_{Leq}(P_i) dP_i$. Although agents from origin I_L anticipate peer's expected effort, their expected relative deprivation is affected by the expected effort of agents from I_U (leader) and the heterogeneity of P_i. First, regardless of e_{Leq}^{b}, the predictions are consistent with "the self-fulfilling beliefs model" in the particular case where every agent from lower-class background compares herself only with agents of the same origin ($P_i = 0$). Namely when agents share public beliefs and assume ex-ante that they belong to a reference group whose members are all I_L, they adopt a behavior that validates their reference group expectations (if $P_i = 0$ for all agent $i \in I_L$ then, $RD(0, e_U^b, e_L^b \mid I_L)$=0 and $e_{Leq}(0) = e_{Ueq}$). When the structure of reference groups is heterogeneous $P_i \neq 0$, observe that $RD(P_i, e_U^b, e_L^b \mid I_L)$=1 and $e_{Leq}(P_i) \neq e_{Ueq}$. Under conditions I and II, agents from lower-class origin always have incentives to assume strategies to improve their opportunities to achieve a better life. As a result, $e_{Ueq} < e_{Leq}(P_i)$ if $P_i \neq 0$. Furthermore, conditions IV and V establish greater incentives. However, under conditions III and VI, $e_{Ueq} > e_{Leq}(P_i)$ if $P_i \neq 0$. Namely, expected relative deprivation discourages agents from low social origin and they choose low effort.

Finally, these results assume the two-step decision process, but general predictions for income mobility do not change if we assume a simple-step process. In this case, agents do not care about ex-ante relative deprivation and agents directly maximize expected utility taking the reference group income as given. We focus now on effort decisions of agents from I_L and to simplify this discussion, we incorporate the following additional informational assumption, which does not change the results.

- *(IA.vi)* We incorporate an upper bound for the expected effort of the agents I_U, $e_U^b < e_{Ueq}^*$.

For agents from origin I_U, the equilibrium is $e_{Ueq} = e_{Ueq}^* = e_{Ueq}^b$. For agents from origin I_L the equilibrium is defined by $e_{Leq}(P_i) > (1-\alpha)\theta\beta_M\Delta y$ if $F(P_i) \neq 0$ (namely at least one agent $i \in I_L$ compares with agents from I_U origin). Observe that in this case, $e_{Leq}^b = \int_0^1 F(P_i)e_{Leq}(P_i)dP_i > (1-\alpha)a\theta\beta_M\Delta y = e_{Ueq}^b$. As a result, under conditions I or II (IV or V), reference group income always motives (discourages) higher optimal effort of agents from low social origins when there is heterogeneity in the composition of their reference groups. Furthermore, only if $P_i = 0\,\forall i \in I_L$, $e_{Leq} = (1-\alpha)a\theta\beta_M\Delta y = e_{Ueq}$, which confirms self-fulfilling-belief.[28]

3.8 Efficiency implications

We next explore the properties of an equilibrium where individuals exert effort as described in Proposition 2 above and take others' choices as given (i.e. Cournot-Nash equilibrium). Assume a continuum of agents from origin I_L, who differ by the composition of reference group (P_i), with density function F(P_i). That is, we keep *assumption IA.vi* (presented in the previous section) and for notational simplicity we omit subscript i.

[28] Observe that contrary to results based on two-step process, in this case, expected peers' effort may encourage effort of agents from origin I_L, even if $P = 0$.

For agents from upper-class social origins $E(y^R \mid I_U) \geq 0$, so equilibrium effort level is exogenous (see section 3.5). Under these conditions, and considering equations (12) and (12.b), the expected social welfare is given by,

$$\begin{gathered} W(e_u, e_L(P), e_u^b, e_L^b) = E(U \mid I_U) + w \int E(U \mid I_L) = \\ (1-\alpha)\Delta y(\theta\beta_M e_U + \Delta\pi + \pi) - \tfrac{e_U^2}{2c} + \\ n\int_0^1 \left[(1-\alpha)\Delta y(\theta\beta_M e_L(P) + \pi) + \alpha G(y^R(P), e_L(P)) - \tfrac{e_L(P)^2}{2c}\right] F(P)dP \end{gathered} \tag{18}$$

where the number of agents from origin I_U is normalized to unity, and $w > 0$ represents the number of agents I_L for each agent from origin I_U. Under perfect information the expected effort equilibrium is $e_{Ueq}^b = e_{Ueq}$ and $e_{Leq}^b = \int_0^1 F(P)e_{Leq}(P)dP$.

For society to be at an optimum,

$$e_{Ueq\,opt} = c\left[(1-\alpha)\theta\beta_M\Delta y - \lambda_1\right] \tag{19}$$

$$e_{Leq\,opt}(P) = c\left[(1-\alpha)\theta\beta_M\Delta y + \alpha\theta\beta_M\Delta y G_{y^R}(y^R(P), e_L(P)) + \alpha G_e(y^R(P), e_L(P)) - \frac{\lambda_2}{w}\right] \tag{20}$$

$$\lambda_1 = -w\alpha\theta\beta_M\Delta y\int_0^1 \left[F(P)PG_{y^R}(y^R(P), e_L(P))dP\right] \tag{21}$$

$$\lambda_2 = w\int_0^1 \left[F(P)(1-P)\alpha\theta\beta_M\Delta y G_{y^R}(y^R(P), e_L(P))dP\right] \tag{22}$$

$$e_{Ueq\,opt} - e_{Ueq\,opt}^b = 0 \tag{23}$$

$$\int_0^1 F(P)e_{Leq\,opt}(P)dP - e_{Leq\,opt}^b = 0 \tag{24}$$

where λ_1 and λ_2 are the multipliers on constrains defined in equations (23) and (24) respectively. Therefore, if we compare equations (20) and (19) with the previous equations (8) and $(8.b)$ for private effort choices, the expected equilibrium is not optimal. Due to the concavity of (18) and to λ_1 and λ_2 being positive (from eq. (21) and (22)), socially expected desirable levels of effort are below those which agents make individually. This is because effort decisions affect the relative deprivation of others and also because of the well-known 'rat-race' effect induced by relative concern. Since agents ignore the externalities stemming from their decisions, the equilibrium based on individual decisions is suboptimal. This result is in accordance with the findings from economic models where individual utility depends on relative situations (Clark and Oswald, 1998; Piketty, 1998; Frank, 1997; 2005).

Unlike previous models, however, ours distinguishes two possible sources of externalities. On the one hand, eq. (19) shows that the effort of agents from origin I_U (leaders) generates a negative externality on the decisions of agents from origin I_L(followers). Furthermore, this externality "between" social origins, will be higher the higher $\Delta\pi$. That is, regardless of the effort decisions of agents from origin I_L, lower ex-ante inequality reduces expected inefficiency.[29] On the other hand, there is a "within externality", which comes from the effort decisions of peers from origin I_L.

Finally, note that since inefficiency results from individuals exerting too much effort, the presence of positional discouraged agents (whose reaction to relative deprivation is exerting lower effort) will decrease inefficiency –as well as expected upward income mobility of agents from lower social origins, I_L.

4 A model of effort choice with reference group and intertemporal learning

The results of the previous section could be interpreted as a benchmark, which considers a situation in which there is perfect information (expected effort is known, constant and exogenous) or fully forward-looking agents ($IA.v$). Now we assume that agents from origin I_L do no know the effort of the peers of their generation and they choose their effort based on their beliefs (e_L^{b}). Each generation updates their beliefs with respect to the previous generations' beliefs by trial and error methods using local knowledge based on their peers' past experience. Beliefs are updated by a backward-looking learning process, that is, in light of the recent experience of peers from the same social origin from a previous generation. This establishes a connection between expected peers' effort and performance in terms of the income mobility of a previous generation. Bowles (2004) argues that the backward-looking learning approach has advantages when compared to the forward-looking learning process.[30] We assume that agents incorporate information of the economic performance of the previous generation when they update their *a priori*

[29] Observe that equilibrium based on individual decisions will be optimum when $P_i = 0$. But in this case, effort decisions reduce income mobility. Furthermore, aggregate inefficiency will be higher when more agents from origin I_L present high reference income ($(F(P_i)' > 0$).

[30] Bowles (2004) considers backward-looking learning process inside evolutionary game theory. In contrast to the forward-looking agents in classical game theory, this approach addresses the history of the agents.

public beliefs, which are transmitted from previous generations.[31] Finally, as we discuss in section 4.4, this learning process seems useful to explain the formation of aspirations based on social interactions (Appadurai, 2004, Genicot and Ray, 2017).

Our model establishes leader-follower dynamics between agents from origin I_L and agents from origin I_U. Therefore, in order to analyze the role of the reference group as a determinant of income inequality persistence, we can retain assumption $IA.vi$ used in section 3.7 ($e_U^b < e_{Ueq}^*$ and it is exogenous) because agents from origin I_L can not affect the effort decision of agents from I_U. This implies that the optimal effort of agents from origin I_U is $e_{Ueq} = (1-\alpha)c\theta\beta_M\Delta y$, which represents a benchmark for agents from origin I_L. In the remainder of this section, we focus on agents from origin I_L (for notational simplicity we omit the social origin sub-index L and U for the rest of this section).

4.1 The information structure

We remove assumption $IA.v$ and now agents are uncertain about the real effort of their peers when they choose their effort level. Each agent takes others' effort as given within the same period, but they update their beliefs about e^b between generations. Informational assumptions $IA.i$ and $IA.ii$ from section 3 remain the same, but we include the following additional informational assumptions:

- $IA.vii$. Individual effort levels are not publicly observable, but agents know that they are between a certain "high effort level" ($\overline{e} \leq \bar{E}$) and a certain "low effort level" ($\underline{e} \geq 0$), with ($\overline{e} \geq \underline{e}$). Observe that $IA.vii$ substitutes assumption $IA.iii$.
- $IA.viii$. The current generation knows the social mobility experienced by the previous generation, which represents a signal of their effort levels. Namely, they know ${x'}_t$ which is the real share of successful agents from origin I_L from generation t.
- $IA.ix$. Public beliefs about effort are transmitted across generations, therefore, generation $(t+1)$ has *a priori* information ($\mu_{t+1}^{apriori}$) based on the real beliefs of generation t (μ_t).
- $IA.x$. Based on the signals x'_t, generation $(t+1)$ updates their *a priori* beliefs ($\mu_{t+1}^{apriori}$) about the effort of their peers.

As agents know π, θ, β_M, and Δy, given e_{M_t} they know the distribution of signals (*IA.i, IA.ii, IA.vi, IA.vii, IA.viii IA.ix*), which describes the expected share of successful agents

[31]Other papers have used this learning procedure and they place an emphasis on the information transmission between generations and the significance of past trajectories in order to explain heterogeneous beliefs equilibrium. Piketty (1995) used Bayesian learning to update the belief about the parameters of the economy, Piketty (1998) used it to explain the public beliefs about status, and Breen and García-Peñalosa (2002) used it to describe the difference in preferences across genders.

from origin I_L from generation t, conditional on the state v'_t. Taking x_t as the real share of successful agents from origin I_L from generation t, agents can derive the probability of the signal $x_t = x'_t$, conditional on the state being v'_t and their a priori beliefs:

$$Pr(x_t = x'_t \mid \nu'_t) = \Omega(\varepsilon'_t, \nu'_t \mid \nu'_t) = \Phi(e_{M_t}(\varepsilon'_t), \nu'_t \mid \nu'_t) \tag{25}$$

where ε_t and ν_t are vectors of n dimensions, which respectively reflect individual efforts in t $(e_{1t}, e_{2t}.., e_{nt}) = \varepsilon_t$ and n random variables $(v_{1t}, v_{2t}.., v_{nt})=\nu_t$, and ε'_t and ν'_t are particular realizations of both vectors. For notational simplicity, we introduce the function $\Phi(.)$, whose argument is mean effort of agents from origin I_L at t, (e_{M_t}), which is a linear function of each element in the vector ε_t. A detailed description of these derivations and results is presented in Appendix B.

4.2 Intergenerational learning

Given assumption *IA.vi*, the expected effort for their peers in generation t is defined as,

$$e^b_t = \mu_t \overline{e} + (1 - \mu_t)\underline{e} \tag{26}$$

where μ_t is the public belief of generation t about the participation of high effort agents among economically successful agents from the previous generation from origin I_L. This parameter is interpreted as the subjective probability attached by the entire generation that $\overline{e}$ was the effort of successfully agents from origin I_L.

Each generation $(t+1)$ of agents from origin I_L observes a signal x'_t from the previous generation. As a result, they know the real percentage of economically successful agents from origin I_L in the previous period, but they do not observe which one of them made high or low effort $(IA.ii)$.

According to assumption *IA.viii*, the current generation have *a priori* beliefs $\mu^{apriori}_{t+1} = \mu_t$ (Note that $e^{apriori}_{t+1} = e^b_t = \mu_t\overline{e}+(1-\mu_t)\underline{e}$. Since mobility performance is only stochastically related to effort, "evaluation errors" may occur. If $x'_t \neq \Phi(e^{apriori}_{t+1}, \nu'_t \mid \nu'_t, \mu_t)$, agents believe that there is an error in their *apriori* beliefs $(\mu^{apriori}_{t+1})$. As a result, based on the signals x'_t generation $(t+1)$ updates their *a priori* beliefs about the effort of their peers according to Bayes' rule.

Observe that the importance of those errors depends on the correlation between e_{it} and v_{it}. On the one hand, when $\overline{\sigma} = Corr(e_{it}, v_{it}) > 0$ the shock does not "redistribute" economically successful agents between low and high effort agents. As a result, "effort pays" and high effort agents dominate amongst successful agents. On the other hand, when $\underline{\sigma} = Corr(e_{it}, v_{it}) < 0$, the shock "redistributes" agents, that is, some agents with low effort achieve economic success. Now, although effort has a positive impact on the probability of high income, effort rewards are relatively low compared to former case. As a result, the proportion of low effort agents is relatively high among economically successful agents, and then e_t^b (and μ_t) should be lower. Observe that the sign of this correlation represents two states of the world.[32] The distribution function of signals depends on the real state of the world. The probability that the public signal x_t' is realized conditional on the state being $\overline{\sigma}$ or $\underline{\sigma}$ is defined as,

$$Pr(x_t = x') = \Phi(e_{M_t}(\varepsilon_t'), \nu_t' \mid \overline{\sigma}, h_{t-1}) = \overline{\Phi}(e_{M_t}(\varepsilon_t'), \nu_t' \mid h_{t-1}) \qquad (27)$$

$$Pr(x_t = x') = \Phi(e_{M_t}(\varepsilon_t'), \nu_t' \mid \underline{\sigma}, h_{t-1}) = \underline{\Phi}(e_{M_t}(\varepsilon_t'), \nu_t' \mid h_{t-1}) \qquad (27b)$$

where h_{t-1} describes the decisions history of all agents I_L from previous generations $(t-1, t-2, ...)$.

As $\mu_t = \mu_{t+1}^{apriori}$ is an *a priori* probability assigned to high effort $\overline{e}$, it also represents the subjective probability of generation $(t+1)$ that $\overline{\sigma}$ is the true state of the world. An individual from generation $(t+1)$ uses mobility results (the signals x_t') to update their inherited *a priori* beliefs ($\mu_t = \mu_{t+1}^{apriori}$) about the effort of their peers.

In sum, the sequence of events is as follows:

1. Agents from origin I_L from generation t decide how much effort to exert, based on their beliefs about expected effort of their peers in the current generation (e_t^b).
2. After the realization of ν_t, some of them obtain y_0 while others obtain y_1 (which generates the public signal x_t').
3. The belief of generation t (μ_t) is inherited by the next generation ($\mu_{t+1}^{apriori} = \mu_t$).
4. The updated belief of generation $(t+1)$ (e_{t+1}^b) combines that *a priori* information with the mobility outcome of generation t. Once mobility of generation t is realized, the next generation updates their *a priori* beliefs and they choose their effort level based on their updated beliefs.

4.3 The long term equilibrium distribution of beliefs

Following Piketty (1995, 1998) and Breen and García-Peñalosa (2002), these assumptions imply that intergenerational learning takes the form of Bayesian updating, with beliefs being updated by the current generation from the previous generations. Bayesian learning implies that the outcomes of the previous generation are interpreted in the light of

[32] Correlation could be interpreted as an expression of the heterogeneous capacity of the agents to respond to different shocks, given their effort.

the *a priori* beliefs. As a result, an effort belief (e^b_{t+1}) combines *a priori* information transmited from previous generations e^b_t and information about the mobility experienced by the previous generation x'_t. The posterior beliefs of the following generation which observe the signal x'_t is given by Bayes' rule,

$$\mu_{t+1} = \frac{Pr(\bar{\sigma}\cap x'_t|h_{t-1})}{Pr(x'_t|h_{t-1})} = \frac{Pr(\bar{\sigma}|h_{t-1}).Pr(x_t=x'_t|\bar{\sigma},h_{t-1})}{Pr(\bar{\sigma}|h_{t-1})Pr(x_t=x'_t|\bar{\sigma},h_{t-1})+(1-Pr(\bar{\sigma}|h_{t-1}))Pr(x_t=x'_t|\underline{\sigma},h_{t-1})} \\ = \frac{\mu_t Pr(x_t=x'_t|\bar{\sigma},h_t)}{\mu_t Pr(x_t=x'_t|\bar{\sigma},h_{t-1})+(1-\mu_t)Pr(x_t=x'_t|\underline{\sigma},h_{t-1})} \tag{28}$$

where the *a priori* belief $\mu^{apriori}_{t+1}$ is equal to μ_t, and the terms $Pr(x_t = x'_t \mid \bar{\sigma}, h_{t-1})$ represent the conditional probability of the public signals x'_t given that h_{t-1} occurs and that the true state is $\overline{\sigma}$. These probabilities were defined when we introduced the distribution function of signals (eqs. (27) and (27b)). Under these informational assumptions, agents know the functions of the distribution of signals (see Appendix B), so by replacing them in eq. (28) we arrive at the following expression,

$$\mu_{t+1} = \frac{\mu_t\overline{\Phi}(e^b_t, \nu'_t \mid h_{t-1})}{\mu_t\overline{\Phi}(e^b_t, \nu'_t \mid h_{t-1}) + (1-\mu_t)\underline{\Phi}(e^b_t, \nu'_t \mid h_{t-1})} \tag{28b}$$

This function describes the evolution of a generations' beliefs over time. Note that this function depends on *a priori* beliefs. As a result, when there are heterogeneous *a priori* beliefs, the same mobility outcome can give rise to different posterior beliefs. If we consider equations (26) and (28b) together, the effort beliefs are updated according the following rule,

$$\begin{cases} \overline{\Phi}(e^b_t, \nu'_t \mid h_{t-1}) > \underline{\Phi}(e^b_t, \nu'_t \mid h_{t-1}) & \Longleftrightarrow \mu_{t+1} > \mu_t & \Longleftrightarrow e^b_{t+1} > e^b_t \\ \overline{\Phi}(e^b_t, \nu'_t \mid h_{t-1}) < \underline{\Phi} e^b_t, \nu'_t \mid h_{t-1}) & \Longleftrightarrow \mu_{t+1} < \mu_t & \Longleftrightarrow e^b_{t+1} < e^b_t \end{cases} \tag{29}$$

Whether the updated weight placed on $\bar{e}$ is greater than the *a priori* probability depends on whether, for the level of effort chosen by the previous generation, the signal observed is more likely to have occurred for $\bar{\sigma}$ than for $\underline{\sigma}$. If a generation t experienced a relatively high mobility outcome with respect to his *a priori* beliefs, the conditional probability of this event given previous history h_{t-1}, is greater for $\bar{\sigma}$ than for $\underline{\sigma}$. As such generation $t+1$ places a higher weight on $\bar{e}$. The opposite holds for the case of low mobility results. The rationality of the updating belief rule is the following: when agents of generation $(t+1)$ have an *a priori* belief that their peers had made a high effort but were not rewarded with upward mobility, there will be some downward adjustment of the expected effort for their current peers.

A general property of this form of Bayesian learning is that the stochastic process μ_t describes a martingale, what generation t expects its successors to know next period is exactly what generation t knows today. Namely, agents' best guess in generation $(t+1)$, as to their posterior belief in any later period is their posterior belief in period t, namely

$E(\mu_{t+m} \mid \mu_t, h_t)$ with $m > 1$ (Aghion et al., 1991; Piketty, 1995; Smith and Sørensen, 2000). As a result, $E(e^b_{t+m} \mid \mu_t, h_{t-1}) = E(\mu^b_{t+m} \mid \mu_t, h_{t-1})\bar{e} + (1 - E(\mu^b_{t+m} \mid \mu_t, h_{t-1}))\underline{e} = e^b_t$. Assume, without loss of generality, that the true state of the world is $\bar{\sigma}$ (namely "effort always pays").[33] Therefore $\mu_t = 1$ is equivalent to allocating full weight to the truth. Pick $\overline{\sigma} \neq \underline{\sigma}$, with $\mu(e^b_{t-1}, \overline{\sigma}, \nu_t) > 0$, and define for any $t > 1$ the likelihood of $I_t = \frac{\mu(e^b_{t-1}, \overline{\sigma}, \nu_t)}{\mu(e^b_{t-1}, \underline{\sigma}, \nu_t)}$, which follows a stochastic process $\{\mu_t\}$. It describes a martingale conditional on the true state of the world ($\bar{\sigma}$). As a result, standard martingale convergence results can be applied (Aghion et al., 1991; Piketty, 1995; Smith and Sørensen, 2000; Breen and García-Peñalosa, 2000). Piketty (1995) and Breen and García-Peñalosa (2000) derived three propositions about this process, which could be interpreted in terms of our learning process.

1. The martingale convergence theorem implies that the likelihood ratio, and hence beliefs, converge in the long term. For any initial beliefs, μ_0, in the long term beliefs converge toward some stationary beliefs, μ_∞ with a probability of one. Therefore, there is a stable solution about the level of expected effort, which is defined as, $e^b_\infty = \mu_\infty \overline{e} + (1 - \mu_\infty)\underline{e}$.

2. Given the true state of the world $\bar{\sigma}$, the Bayesian updating function defined in eq. (28) has three fixed points. One of them is not stable $\mu_{1\infty} = 0$. There are two stable long term equilibrium beliefs, one is an interior fixed point $\mu_{2\infty} > 0$ and the other is a corner solution $\mu_{3\infty} = 1$.

3. If initial beliefs are $\mu_0 < \mu_{2\infty}$, then it converges to $\mu_{2\infty}$ with a probability of one. As a result, $e^b_{2\infty} = \mu_{2\infty}\overline{e} + (1 - \mu_{2\infty})\underline{e}$. In contrast, if initial beliefs are higher than $\mu_{2\infty}$, then they will be attracted with positive probability $Pr(\mu, \mu_{2\infty})$ by $\mu_{3\infty} = 1$, and with positive probability $[1 - Pr(\mu, \mu_{2\infty})]$ by $\mu_{2\infty}$.

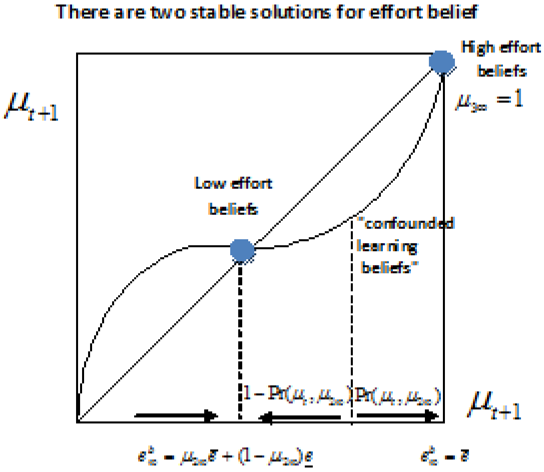
Observe that both stationary beliefs allocate a positive weight to the true state of the world. In terms of effort beliefs, $e^b_\infty = \underline{e}$ is not a stable solution, while $e^b_{2\infty} = \mu_{2\infty}\overline{e} + (1 - \mu_{2\infty})\underline{e}$ and $e^b_{3\infty} = \overline{e}$ are stable solutions. Finally, when the interior solution $\mu_{2\infty}$ holds,

$$\overline{\Phi}(e^b_t(\mu_{2\infty}), \nu'_t \mid h_{t-1}) = \underline{\Phi}(e^b_t(\mu_{2\infty}), \nu'_t \mid h_{t-1}) \Longleftrightarrow \mu_{t+1} = \mu_t \Longleftrightarrow e^b_{t+1} = e^b_t. \tag{30}$$

This expression implies that when agents hold the *a priori* belief $\mu_{2\infty}$, the resulting expected probability is the same under $\bar{\sigma}$ or $\underline{\sigma}$. Breen and García-Peñalosa (2002) named $\mu_{2\infty}$ as "confounded learning beliefs". At this point nothing can be learned from the previous generations' signals, and *a priori* beliefs are equal to the posterior beliefs. They demonstrated that the probability of converging to the true belief is given by

[33] Piketty (1995) discusses extensively the reasons that justify such assumption.

Figure 2: **The equilibrium of beliefs**



$$Pr(\mu, \mu_{2\infty}) = \frac{\mu_0 - \mu_{2\infty}}{\mu_0(1 - \mu_{2\infty})} \tag{31}$$

As a result, long term equilibrium beliefs depend on the initial beliefs and the quality of the public signal information. This result is due to the fact that the same mobility outcome can give rise to different posterior beliefs depending on the probabilities initially attributed to each situation. Successive learning across generations may be complete, as a result, generations will access the true value of σ, $\overline{\sigma}$. In this case, an equilibrium belief about the expected effort of an agent from origin I_L is $e^b_{3\infty} = \overline{e}$. Namely, in this case "effort always pays" in the long term, and agents from origin I_L expect their peers to exert a high level of effort. One point worth noting here is that $\overline{e}$ may not be the "true" mean effort of agents from origin I_L. This expected level of effort is the most likely value given that $\overline{\sigma}$ is the true state of the world, h_{t-1} the history of generations from social origins I_L, and μ_0 the initial beliefs. In other words, evidence shows that effort pays in terms of mobility, and that successive learning across generations leads to the highest expected effort. However, the learning process across generations may be incomplete. In this case agents perceive that effort rewards are relatively low, even if this is not true. As a result, agents place a strictly positive weight on the true state of the world ($\overline{\sigma}$), and long term equilibrium of the expected effort $e^b_{2\infty}$ is lower than $\overline{e}$, but is higher than $\underline{e}$. Although "effort pays" and promotes high income mobility, initial beliefs and mobility trajectories lead, in the long term, to relative lower expected effort for agents from origin I_L.

Equilibrium with intergenerational learning and self-encouraged agents

In this section we assume that all agents are identical and are self-motivated (Conditions I and V). Recall that these conditions assume that agents get utility from effort and thus relative deprivation always motivates high effort. This allows us to explore how effort decisions are affected by income mobility of previous generations and expected relative deprivation (results under alternative assumptions are discussed in the next section).

Under imperfect information, social origin establishes a relationship between generations in two ways. On the one hand, the probability of economic success depends on social origin (direct channel). On the other hand, there is an indirect channel, because the experienced mobility of previous generations affects the beliefs about peers' expected effort. Equation (29) provides a rational updating process, where history is important in determining equilibrium beliefs and public expected effort. Both channels determine the incidence of reference groups through their expected relative deprivation.

In the steady state, the Bayesian learning function leads to social beliefs $e_L^b = e_{L\infty}^b$. Agents from origin I_L and the same P_i will choose the same optimal effort $e_{L\infty}(P_i)$, which is constant, $e_{Leq\,t-1}(P_i) = e_{Leq\,t}(P_i)$, $e_{t-1}^b = e_t^b$. Considering $E(y_\infty^R \mid I_L) = \xi(e_{Leq}(P), e_{L\infty}^b, \overline{e_U^b}, P)$ in eq. (8), we arrive at the following expression,

$$e_{L\infty}(P_i) = \begin{cases} e_{L\infty}^* = (1-\alpha)a\theta\beta_M\Delta y & if\, E(y_\infty^R \mid I_L) \geq 0 \\ e_{L\infty}^{**} = e_{L\infty}^* - c\theta\beta_M\Delta y G_{y_\infty^R}() - c\alpha G_e() & if\, E(y_\infty^R \mid I_L) < 0 \,\&\, e_{L\infty}^{**} < E \\ e_{L\infty} = E & if\, e_{L\infty}^{**} \geq E \end{cases} \tag{32}$$

As a result, for self-stimulated agents from origin I_L the model predicts two possible scenarios about effort level in the long term. First, when $\mu_0 > \mu_{2\infty}$ agents' beliefs will be attracted with probability $Pr(\mu, \mu_{2\infty})$ by $e_{L\infty}^b = \overline{e}$. Regardless of P_i , agents from origin I_L tend to choose higher effort levels than agents from origin I_U, because expected effort of their peers is high. Expectation of peer's effort will increase, and so will individual effort in the future. Agents with higher P choose high effort because their relative deprivation and reference income are relatively high. They are stimulated by the expected income of agents from origin I_U but also by the high effort of their peers from origin I_L. Steady-state effort will always be equal or larger for agents with higher P_i, as they include more agents from origin I_U in their reference group. Finally, observe that when $P_i = 0$ results are consistent with the "self-fulfilling beliefs" of Piketty's model.

On the other hand, due to the initial condition and the past trajectories of the previous generation of agents from origin I_L, the long-term social belief could be $e_{2\infty}^b$. Now, expected effort for agents from origin I_L is relatively low and reference group income will be low if P is low. Relative deprivation leads to lower long-term effort level than in the previous case, and its level will be more similar to the effort of agents from origin I_U. Observe that now there are two possible dynamics. On one hand, when $\mu_0 < \mu_{2\infty}$, expected effort will increase, and so will individual effort in the future. However, those optimist beliefs have a threshold and steady-state effort beliefs will be relatively low. On the other hand, when $\mu_0 > \mu_{2\infty}$, agents will be attracted with probability $1 - Pr(\mu, \mu_{2\infty})$ by $\mu_{2\infty}$(and $e^b_{2\infty}$). Since agents believe that their peers (all agents are I_L) in the reference group will decrease their effort, their reference income will be lower (relative income effect is lower) and they will choose a lower effort level. This situation determines "self-fulfilling beliefs" due to effort beliefs.

When the learning function leads to social belief $e^b_{2\infty}$, the reference group effect reduces income mobility. Furthermore, **if as** in section 4.3 we assume that$\overline{\sigma}$ is the true state of the world, the lower effort level for agents from origin I_L would be suboptimal in the long term. Although "relative effort pays" and promotes high income mobility, agents from origin I_L are inefficiently discouraged from trying to move up, due to social beliefs, mobility trajectories, and inequality.

Equilibrium with intergenerational learning and no self-encouraged agents

Conditions III, IV, VI, and VII, assume that relative effort is a cost, which establishes an ambiguous relationship between effort and reference group income. We cannot really say in general whether reference groups reduce or amplify inequality persistence of economic success between agents from different social origins. However, the model allows us to discuss some interesting issues. The intuition is that relative deprivation encourages agents up to a certain point, but beyond that, relative deprivation discourages them (*Positional discouraged agent*). To be more concrete, let us assume that there is a y^{RG*} such that conditions III and VI hold if $y^{RG} < y^{RG*}$, while conditions IV and VII hold if $y^{RG} \geq y^{RG*}$ (*Positional stimulated agent*).

Case 1. If $P_i = 1$, expected peer effort does not matter. Reference group leads to higher effort only if $E(y_i \mid I_U) < y^{RG*}$. Conversely, if $E(y_i \mid I_U) \geq y^{RG*}$ ex-ante inequality leads to a low effort trap.

Case 2. When $P_i \neq 0$ there is an expected peer effort level $e^{b*}_{\infty}(P_i) = \frac{y^{RG*} - y_0 - \pi \Delta y - P_i(\Delta \pi \Delta y - \theta \beta_M e^b_U)}{(1-P_i)\theta \beta_M}$, which determines critically the long-term effort equilibrium for agents from I_L. By following an reasoning analogous to the previous section's, we arrive to a long-term effort level e^b_{∞}. Under these assumptions, higher expected effort of agents from origin I_L and P_i leads to higher steady-state effort, $e_{L\infty}(P_i)$, only if $e^b_{L\infty} < e^{b*}_{\infty}(P_i)$. Thus, the conclusions obtained in the previous section remain unchanged. However, if $(e^b_{L\infty}) > e^{b*}_{\infty}(P_i)$ a higher expected effort of agents from origin I_L and P_i discourages their long-term effort, whose level will be lower than the equilibrium effort level presented in section 4.3. Observe that under these assumptions the composition of reference groups is even more important for social mobility.

Let us focus now on the role of ex-ante inequality between social origins, which was measured by $\Delta\pi$. In both Case 1 and 2, there is a non-linear (inverted-U shaped) relationship between ex ante inequality and long-term effort level of agents from origin I_L. Also, when $P_i \neq 0$ and a given level y^{RG*}, there is a trade-off between $e^b_\infty(P_i)$ and $\Delta\pi$. When $\Delta\pi$ is low, a positive relationship between expected peer effort $e^{b*}_\infty(P_i)$ and effort $e_{L\infty}$ is quite feasible. Higher peer effort encourages agents from low social origin, because they perceive that expected relative rewards are high compared to the relative costs of effort. However, when $\Delta\pi$ is high, $e^{b*}_\infty(P_i)$ will be low. As a result, though expected peer effort will be high, due to ex-ante inequality, agents from social origin I_L reduce their effort level in order to avoid frustration. Now, ex-ante inequality and high expected peer effort lead to lower long-term effort equilibrium, as compared with section 4.3.

4.4 Reference groups and aspiration failure

In this section we use our model to explain aspiration failure proposed by Ray (2006). Genicot and Ray (2017) argue that the formation of aspirations is one of the most relevant factors in explaining upward mobility. They define aspiration as a realistic and attainable target, which, ex-ante, is beyond an agent's possibilities, but which are potentially achievable. They emphasize the role of social interactions and assume that aspirations are based on the current and past achievements of an agent's socioeconomic neighborhood, which is located within some exogenously given social window ("aspiration window"), defined as $\psi(y_i, D(y_i))$. As a result, an agent's aspirations are determined by her income and the distribution of wealth $(D(y_i))$ in her cognitive window, which could include her peers or individuals far richer than her. As a result aspiration formation is defined as $a : a(\psi(y_i, D(y_i)))$. Then, they assume that an agent's objective function considers the "aspiration gap" $(ag = y - a)$, namely the income difference between her income and her economic aspiration.

$$U(y_i, ag_i) = U(y_i, G(y_i - a(\psi(y_i, D(y_i))) \tag{33}$$

Based on these ideas, Ray (2006) identifies two types of aspiration failure. Aspiration failure type I occurs when agents from low social origin do not include agents from high social origin in their aspiration window. As a result, the aspirations gap is low, as will be individual investments for the future. In aspiration failure type II, agents from low social origin include individuals from richer origins in their aspiration window, but the previous inequality and the relative costs of effort are so high that agents perceive the goal to be unattainable and they are discouraged. As a result they reduce their aspirations and investment level in order to avoid frustration.

If we define the income aspiration as the income reference group ($a = y^{RG}$), we can discuss the conditions that lead to these types of aspiration failure. Observe that P represents the bandwidth of the aspiration window and it provides heterogeneity in reference group income. Second, because our model assumes agents from two social origins, it is useful to analyze how aspirations are socially determined. Furthermore, Ray (2006) argues that an aspiration window depends on how much perceived mobility there is in society, the higher the extent of mobility, the broader the aspirations window. The intergenerational learning proposed in section 4.3 seems adequate to deal with this issue.

Previous papers focus on the effect of aspirations on income growth, wealth distribution, preferences for redistribution, or income mobility (Stark, 2006; Bogliacino and Ortoleva, 2011; Genicot and Ray, 2017; Besley, 2017). Our model has more micro focus than these papers. Dalton et al., (2016)'s model has a micro perspective and focuses on constraints internal to individuals which leads to behavioral poverty traps. Our paper focuses on social constraints and how they lead to low mobility traps.

On the one hand, when individuals are self-motivated, a very low P represents a restricted aspiration window, which leads to aspirations failure type I. In this case, the expected aspiration gap is low, and agents from origin I_L are not encouraged to increase their effort. This will especially be the case if there is economic polarization or other forms of stratification.

On the other hand, there is aspiration failure type II when individuals from I_L include individuals I_U in their "aspiration window" (high P). Failure type II seems less consistent with "self-motivated" individuals, although when $P \neq 1$, a low $e^b_{2\infty}$ would reduce the effort of agents from origin I_L. When effort beliefs of agents from origin I_L are low, expected "relative deprivation" will be lower, which induces reducing effort. Although "relative effort pays", agents from origin I_L reduce their effort because they believe that their peers in the reference group will decrease their effort. Therefore, expected mobility is low (peer effort "does not pay"), and the aspiration gap leads to a lower long-term effort level compared to those agents with $P = 1$ or a situation with $e^b_{L\infty} = \bar{e}$. This effect will be higher if P is low, which is related to failure type I.

When individuals are not self-motivated, reference groups may directly explain failure type II. First, strong ex-ante inequality between agents from different social origins would lead to lower effort. In this case the relatively poor individuals do aspire to be like the rich, but the income gap is simply too large (see section 4.3). The cost of effort (or investment) is too high, and the reward (in terms of a relative narrowing of the aspiration gap) too low. The reference group leads to aspirations, but the feeling is widespread that such aspirations are largely unreachable. Second, when leisure and relative income are not complements, an agent from social origin I_L is more easily satisfied with her performance and less motivated to achieve high income positions than agents with a less demanding reference group or upper-class origin. As a result, higher reference group income leads to lower effort because agents perceive the goal to be unattainable. Therefore, high relative deprivation reduces the agent's income aspirations and effort level in order to avoid frustration.

5 Conclusion

Our model shows how sociocultural inequalities, in general, and reference groups, in particular, shape inequality persistence. Expected relative deprivation with respect to a reference group determines optimal effort decisions, which is a key determinant of intergenerational income mobility. We identify the conditions under which reference groups leads to high and low income mobility. We show that the size and direction of these effects depend on, (a) the direction of income comparisons (i.e. to whom individuals compare, "(P)"; (b) their intensity (i.e. how much do they compare, α and $G(.)$); (c) ex-ante inequality between agents from different social origins, $\Delta\pi$, and relative effort rewards; and (d) the information about their peers and past income mobility.

When the reference group of low-class origin individuals consists only of low-class origin individuals, and their peers' expected effort is low, their reference income is closely aligned to their expected income. Therefore, they have little incentive to increase their effort, relative deprivation will be low, as will their investments for the future. This leads to a "self-fulfilling belief". However, the effect of a low-class reference group composition could be compensated if their peers' expected effort were high.

When agents from low-class origin include individuals from high-social origin in their reference group, their expected income gap is larger. In this case, the impact of relative deprivation on optimal effort is ambiguous, and assumptions about the functional form of relative concerns are key. When relative concern is additive in the utility function, standard assumptions or prospect theory lead to situations where individual's effort response (and income mobility) are very different. The former lead to self-encouraged agents, while the latter describes discouraged agents. When we assume additivity in the utility function away, the incidence of reference groups depends on the sign of two functions, G_e, which describes how effort affects relative deprivation assessment, and G_{ey^R}, which defines whether leisure and relative income are complements or substitutes. If relative income and leisure are complements, the reference group always promotes higher effort levels. Individuals from lower-class backgrounds are self-motivated by a larger income gap and work harder in the pursuit of personal economic success and social ascent. In this case, reference group income promotes high income mobility, a result that is in stark contrast to predictions from other models of inequality based upon self-fulfilling beliefs and fatalistic predictions.

However, if relative income and effort are substitutes, relative deprivation has an ambiguous effect on effort. In this case, the expected income gap between the individual and her reference group may encourage or discourage lower-class agents. Ex-ante inequality and expected relative deprivation are key determinants in explaining that ambiguity. There is an inverted U-shape relationship between long term effort, and P, on the one hand, and ex-ante inequality, on the other hand. If the income gap is due to the expected effort of their peers, high reference income may increase effort and mobility. However, high ex-ante inequality and low relative effort rewards could reduce the effort of low social origin individuals. This situation, which reduces income mobility, is related with aspiration failure type II (Ray, 2006).

As expected reference group income is contextual, its effects depend on how much mobility is perceived. In considering this issue we assume imperfect information and we model beliefs using a Bayesian learning process. There are two stable solutions for effort beliefs that depend on whether individuals from low-class origins choose high or low effort. In the latter case, because individuals from low-class origins believe that their peers in the reference group will reduce their effort, their reference income will be lower and they will choose a lower effort level. This situation determines a "self-fulfilling belief" due to effort beliefs.

Consistent with previous papers, the reference group effect leads to a suboptimal situation. When we assume forward-looking individuals, this inefficiency is explained by the "between" and "within" social origin effects, and it is higher the higher ex-ante inequality, $\Delta\pi$. If we assume backward-looking individuals, results are ambiguous. In this case, even if we assume that "relative effort pays" and promotes high income mobility, agents from low-class origins would be inefficiently discouraged from trying to move up, due to social beliefs, mobility trajectories, and inequality. As a result their economic aspirations would be inefficiently lower.

Overall, our model –and hence our conclusions– are more general than previous models of inequality based on self-fulfilling beliefs and fatalistic predictions. Unlike previous models, we study how the functional form of relative concern, the composition of the reference group, and the way beliefs are formed shape mobility patterns. Importantly, we characterize the conditions under which reference groups may enhance intergenerational mobility, thus extending previous analyses, such as Piketty (1998). Our model also spells out what conditions may originate Ray's (2006) two aspiration failures.

Our model also provides new arguments to the insights that derive from the behavioural approach. For instance, Haushofer and Fehr (2014) and Congdon et al. (2011) suggest that extreme poverty may have psychological consequences, which affect economic behavior and could lead to people being discouraged from making the best mobility-enhancing investments available, contributing to poverty persistence. Our theoretical contribution helps to better understand these issues, by discussing how reference groups and unequal initial conditions may discourage or encourage mobility-enhancing decisions.

Our results provide insights about what policies may enhance mobility. We show that the reference group effect could increase intergenerational mobility in more integrated societies, where economic diversity in the reference groups were large enough and income inequality were relatively low. Thus, policies that increase the economic diversity of reference groups and that seek to reduce segregation may enhance intergenerational mobility. These include affirmative action, public education, convening young people and enrolling them in programs (e.g. school or kindergarten) away from their communities. The first two policies have also been advocated by Ray (2006) to increase low mobility, when this is due to the presence of low economic aspirations in an unequal society, as they help create local, attainable incentives at the lower end of the income distribution, while the latter two have been advocated by Austen-Smith and Fryer (2005) to fight the effects from "acting White". Redistributive policies may also have an impact on mobility if they alter the reference income. Conditional cash transfer programmes, for instance, primarily aimed at reducing poverty, could modify the composition of reference groups, change effort decisions, and thus affect long-term income mobility.

The results of this paper suggest a number of new avenues for empirical research. On the one hand, they provide a theoretical framework to evaluate the reaction of agents empirically, in terms of effort, when their relative situation and rewards change. On the other hand, they describe how relative concern could affect income mobility through the formation of aspirations. One problem of empirical studies on this issue is that they fail to explain the implications of self-selection into reference groups. In our model, we avoid discussing this issue and consider the parameters that define reference group integration to be a random variable. Our model demonstrates that reference groups affect income mobility even in this hypothetical situation. However, a model which focuses on endogenizing reference group choice is a possible direction for future research. A number of important issues remain to be addressed. First, our approach assumes only two social origins, but this model can be extended to a model in which society has multiple-social origins. Second, in our model the possibility of strategic behavior on the part of agents from different social origins or reference groups is ignored. Third, this paper proposes a bayesian updating belief process, but different learning processes could also be considered. Finally, in this paper we consider only one perspective of status, the comparison role of the reference group, but there are other perspectives of relative or positional concern.

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Appendix A The role of effort rewards and ex-ante inequality on relative deprivation and attitude

Discussion

If we assume that relative effort represents a cost ($G_e(\cdot) < 0$), which is the best case study, we are able to examine how the magnitude of effort rewards stimulates (or discourages) agents. Observe that condition VII defines the locus where individuals face relative deprivation, but they do not respond to a change in reference group income ($\partial e^{**}_{Ueq}/\partial y^{RG} = 0$).[34] The locus which defines "indifferent agents" allows us to identify the alternative values of θ and $\triangle\pi$, that depends essentially on the sign of $G_{y^R y^R y^R}(\cdot)$

> **Proposition 4.** When $E(y^R) < 0$, under non additive comparisons, G_{ey^R} constant and asymmetry in the income comparison:
>
> If $G_{y^R y^R y^R}(\cdot) > 0$ (Principle of diminishing transfers in gaps), there is no monotonic relationship between the sign of $\partial e^{**}_{Leq}/\partial y^{RG}$ and θ and it is ambiguous how individuals respond to higher θ (how agents respond to an increase in θ depends on $G_{y^R y^R y^R}(\cdot)$ and $G_{y^R y^R}(\cdot)$).
>
> If $G_{y^R y^R y^R}(\cdot) \leq 0$, regardless of the functional form of $G_{y^R y^R}(\cdot)$ and $G_{y^R y^R y^R}(\cdot)$, and $\triangle\pi$ there is a unique value of θ, $\tilde{\theta}$, such that $\partial e^{**}_{Leq}/\partial y^{RG} < 0$ if $\theta < \tilde{\theta}$ and $\partial e^{**}_{Ueq}/\partial y^{RG} > 0$ if $\theta > \tilde{\theta}$.

Under the principle of diminishing transfers assumption, an increase in θ has a direct positive effect on effort, because it improves expected relative deprivation, but a higher relative income decreases the sensibility of the marginal utility of relative deprivation ($\downarrow G_{y^R y^R}(\cdot)$ because $G_{y^R y^R y^R}(\cdot) > 0$), which reduces the incentive to increase effort. Given these effects in opposite directions, it is ambiguous how individuals respond to higher θ. However, this ambiguity disappears when $G_{y^R y^R y^R}(\cdot) \leq 0$. In this case, an increase in θ increases expected relative deprivation and the sensibility of marginal utility of relative deprivation. Both effects play in the same direction, and effort will increase.

In the latter case, it is useful to examine the relationship between θ and $\Delta\pi$ (effort rewards and ex-ante inequality rewards) when $\partial e^{**}_{Ueq}/\partial y^{RG} = 0$. There is a function $f(G(.), \theta, e_{Leq}, \triangle\pi) : \tilde{f}(\theta, \triangle\pi)$, which defines the set of all values of θ and $\triangle\pi$ where individuals do not respond to changes in reference group incomes. Given previous assumptions, we can conclude that $\frac{\tilde{f}_\theta(\theta,\triangle\pi)}{\tilde{f_{\triangle\pi}}(\theta,\triangle\pi)} > 0$. In this case, higher θ generates incentives to increase effort, which can be compensated with a higher $\triangle\pi$. To make this result a little more concrete, assume two

[34] To simplify, we assume that G_{ey^R} is constant and $e^b_{Ueq} \leq (1-\alpha)c\theta\beta_M \Delta y < e^b_{Leq}$.

economies A and B, with $\tilde{f_A}(\theta_A, \triangle\pi_A) = \tilde{f_B}(\theta_B, \triangle\pi_B)$, but the former presents higher ex-ante inequality ($\triangle\pi_A > \triangle\pi_B$). In order for there to be a stimulated income gap effect on effort decisions, economy A will require higher effort reward levels θ such $\theta > \theta_A > \theta_B$ Namely, there is less stimulation in more unequal economies. The proof of these results is presented in the next section, where we also demonstrate that the sign of $\frac{\tilde{f}_\theta(\theta,\triangle\pi)}{\tilde{f_{\triangle\pi}}(\theta,\triangle\pi)}$ is indeterminate when $G_{y^R y^R y^R}(.) < 0$.

Proofs

To analyze the role of effort rewards and ex-ante inequality on attitudes toward effort we incorporate two simplifying assumptions, G_{ey^R} is constant and $e^b_{Ueq} < e^b_{Leq}$.

"Indifferent agents" holds, $\theta\beta_M\Delta y G_{y^R y^R}(.) + G_{y^R e}(.) = 0 = I(G(.), \theta, e_{Leq}, e_{Ueq}, \triangle\pi)$. Then:

$$\frac{\partial I(\cdot)}{\partial\theta} = d\theta G_{y^R y^R}(\cdot) + \theta G_{y^R y^R y^R}(\cdot)^{\partial y^R}/_{\partial\theta} = 0 \tag{34}$$

Observe that $G_{y^R y^R}(\cdot) < 0$ and ${}^{\partial y^R}/_{\partial\theta} > 0$ (because $e^b_{Ueq} < e^b_{Leq}$). The locus which defines "indifferent agents" depends essentially on the sign and magnitude of $G_{y^R y^R y^R}(\cdot)$.

When $G_{y^R y^R y^R}(\cdot) > 0$ (Principle of diminishing transfers), there is a function $f(G(\cdot), \theta, e_{Leq}, \triangle\pi) : \tilde{\theta}(\theta) = d\theta G_{y^R y^R y^R}(\cdot) = -\theta G_{y^R y^R}(\cdot)^{\partial y^R}/_{\partial\theta}$, which defines the condition that must be met for ${}^{\partial e^{**}_{Ueq}}/_{\partial y^{RG}} = 0$ for alternatives values of parameter θ. Therefore, given $G(\cdot)$ and $\triangle\pi$, ${}^{\partial e^{**}_{Ueq}}/_{\partial y^{RG}} < 0$ if $\theta^{Low} < \tilde{\theta}(\theta)$ and ${}^{\partial e^{**}_{Ueq}}/_{\partial y^{RG}} > 0$ if $\theta^{high} > \tilde{\theta}(\theta)$. Observe that there is no monotonous relationship between the sign of ${}^{\partial e^{**}_{Ueq}}/_{\partial y^{RG}}$ and θ. How agents respond to an increase in θ depends on $G_{y^R y^R y^R}(\cdot)$ and $G_{y^R y^R}(\cdot)$.

When $G_{y^R y^R y^R}(\cdot) \geq 0$, the function is undefined for a range of values of θ and ${}^{\partial y^R}/_{\partial\theta} = 0$ holds only for unique value of θ.

We focus now in the relationship between θ and $\triangle\pi$ when ${}^{\partial y^R}/_{\partial\theta} = 0$. There is a function $f(G(\cdot), \theta, e_{Leq}, \triangle\pi) : \tilde{f}(\theta, \triangle\pi)$, which defines the set of all values of θ and $\triangle\pi$ where individuals do not respond to changes in reference group incomes. The total derivative of the function $I(\cdot)$ with respect θ and $\triangle\pi$, allows us to analyze the sign of the derivatives of $\tilde{f}(\theta, \triangle\pi)$.

${}^{\partial I(\cdot)}/_{\partial\theta} + {}^{\partial I(\cdot)}/_{\partial\triangle\pi} = \partial\theta \left[\beta_M\Delta y G_{y^R y^R y^R}(\cdot) + \theta\beta_M\Delta y G_{y^R y^R}(\cdot)^{dy^R}/_{\partial\theta}\right] +_{(cont)}$

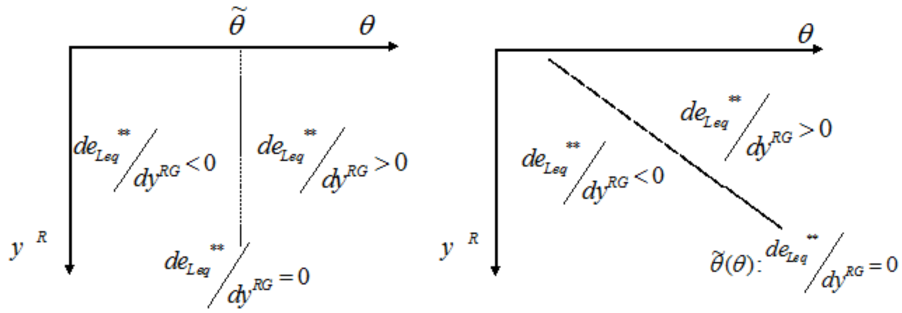
${}_{(cont)}\theta\beta_M\Delta y G_{y^R y^R}(\cdot)^{\partial y^R}/_{\partial\triangle\pi} = 0$

$\partial\theta \left[G_{y^R y^R y^R}(\cdot) + \theta G_{y^R y^R}(\cdot)^{\partial y^R}/_{\partial\theta}\right] = -\theta \left[G_{y^R y^R y^R}(\cdot)^{\partial y^R}/_{\partial\triangle\pi}\right] \partial\triangle\pi$

${}^{\partial\theta}/_{\partial\triangle\pi} = \frac{-\theta G_{y^R y^R y^R}(\cdot)^{\partial y^R}/_{\partial\triangle\pi}}{G_{y^R y^R y^R}(\cdot) + \theta G_{y^R y^R}(\cdot)^{\partial y^R}/_{\partial\theta}}$

Figure A. 1: **The role of effort rewards on relative deprivation and attitude toward effort**

1.$a)$ $G_{y^R y^R y^R}(\cdot) > 0$ 1.$b)$ $G_{y^R y^R y^R}(\cdot) \leq 0$



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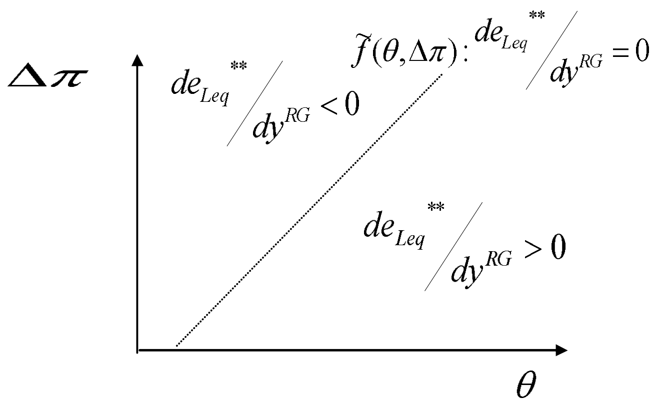
When $G_{y^R y^R y^R}(\cdot) < 0$, both numerator and denominator are positive and ${}^{\partial\theta}/_{\partial\triangle\pi} > 0$ and $\frac{\tilde{f}_\theta(\theta,\triangle\pi)}{f_{\tilde{\triangle}\pi}(\theta,\triangle\pi)} > 0$ (observe that ${}^{\partial y^R}/_{\partial\triangle\pi} < 0$). Namely, to ensure that the condition $\theta\beta_M \Delta y G_{y^R y^R}(\cdot) + G_{y^R e}(\cdot) = 0$ is met, when θ increases, it is necessary a higher relative deprivation (higher $\Delta\pi$) to increases the sensibility of the marginal utility of relative deprivation $G_{y^R y^R}(\cdot)$, and then the marginal utility $G_{y^R}(\cdot)$. The signs of ${}^{\partial\theta}/_{\partial\triangle\pi}$ and $\frac{\tilde{\theta}(\theta,\triangle\pi)}{f_{\tilde{\triangle}\pi}(\theta,\triangle\pi)}$ are undetermined when $G_{y^R y^R y^R}(\cdot) \geq 0$. The increase of θ improves relative deprivation (${}^{\partial y^R}/_{\partial\theta} > 0$), which reduces the marginal utility of relative deprivation ($G_{y^R y^R}(\cdot) < 0$), but at a increasing rate ($G_{y^R y^R y^R}(\cdot) \geq 0$). Given these effects in opposite directions, it is unclear which is the relationship between θ and $\Delta\pi$.

Figure A. 2: **The role of effort rewards and ex-ante inequality on relative deprivation and attitude toward effort ($G_{y^R y^R y^R}(\cdot) < 0$)**



Appendix B The intergenerational learning in detail

Assume that for each agent from origin I_L there is a latent variable which describes the relation between economic success and effort, which is defined in equation (1) as $Y'_{it} = \pi + \theta\beta e_{it}$. An agent i from generation t does not observe e_{jt} of agent j, but he knows the individual social mobility trajectories (y_1 or y_0) of all agents from generation $t-1^{th}(IA.vii)$. For this reason, the mobility outcome of agents I_L from generation t represents a signal about the effort of agents from origin I_L, which contributes to shape the beliefs of generation $(t+1)$.

It is useful to consider that the economic performance is stochastically related to effort, incorporating a random variable v_{it}, which represents the luck of the generation t. Therefore, the expected probability that n agents from origin I_L from generation t reach y_1 is defined as,

$$E(Pr(y_{1t} = y_1, y_{2t} = y_1, ...y_{nt} = y_1 \mid i = 1...n \in I_L)) = \prod_{\forall i \in I} (\pi + \theta\beta_M e_{it} + v_{it}) \tag{35}$$

where v_{it} represents an idiosyncratic shock (which reflects income realization) for each generation t and agent i , with $E(v_{it}) = 0$ and $0 \leq \pi + \theta\beta e_{it} + v_{it} \leq 1$, for $0 \leq e_{it} \leq E$. Observe that once agents i and j from origin I_L choose e_{it} and e_{jt} respectively, $Pr(y_{it} = y_1 \mid i \in I_L)$ and $Pr(y_{jt} = y_1 \mid i \in I_L)$ are two statistically independent events.

Taking x_t as the real share of successful agents from origin I_L from generation t, agents can derive the probability of the signal $x_t = x'_t$, conditional on the state being v'_t,

$$Pr(x_t = x'_t \mid \nu'_t) = \Omega(\varepsilon'_t, \nu'_t \mid \nu'_t) = \Phi(e_{M_t}(\varepsilon'_t), \nu'_t \mid \nu'_t) \tag{36}$$

where ε_t and ν_t are vectors of n dimensions, which respectively reflect individual efforts in t $(e_{1t}, e_{2t}.., e_{nt})$ and n random variables $(v_{1t}, v_{2t}.., v_{nt})$, and ε'_t and ν'_t are particular realizations of both vectors. For notational simplicity, we introduce the function $\Phi(\cdot)$, whose argument is the mean effort of agents from origin I_L in t (e_{M_t}), which is a linear function of each element in the vector ε_t. As agents know π, θ, β_M, and Δy, given e_{M_t} they know the distribution of signals (*IA.i, IA.ii,IA.iv, IA.vi, IA.vii, IA.viii*), which describes the expected share of successful agents from origin I_L from generation t, conditional on the state v'_t.